

Retirement Models

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Genesis

- ◆ **April 2000:**
 - Visit from a financial planner/web site/ “70@65”
- ◆ **Internet stocks**
 - E-Toys: \$10.1b market cap; bankruptcy \$20mm
 - Northpoint communication \$5.6b; to \$135mm
- ◆ **Objective: Practical asset allocation rule**
- ◆ **Multi-period consumption-investment paradigm**
 - Dynamic programming with a constant opportunity set
 - Constant asset allocation (Samuelson/Merton)
 - “it is an exact theorem that investment horizons have no effect on your portfolio proportions.”

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Consider these guidelines for asset allocation, adjusting the percentages for your personal risk tolerance and investment horizon:

- If you are in your 20s or 30s, allocate 70 percent of your investments to aggressive growth or blue chip stocks, and 30 percent to a combination of bonds and money market funds.
- In your 40s, allocate about 50 percent to stocks and 50 percent to a combination of bonds and money market funds.
- In your 50s, allocate 30 percent to stocks and 70 percent to a combination of bonds and money market funds.
- In your 60s, allocate 20 percent to stocks and 80 percent to a combination of bonds and money market funds.

As you can see, you gradually reallocate more to bonds and cash, as you get older. When you're young, you can afford to take more risk that comes with investing in stocks. As you get older, principal preservation—the elimination of any chance of a loss of your investment—becomes more important, as you have fewer opportunities and time to replace it.

Precise allocation among these three categories depends on variables such as the change in interest rates, relative prices and returns of stocks and bonds, and the outlook for economic growth. For instance, if interest rates are falling, you may wish to reallocate more to stocks than you normally would. In general, lower interest rates improve stock returns and hurt returns of bonds and money market funds. Higher interest rates work the opposite way: stock returns decline and the interest rates paid on bonds and money market funds rise.

<http://www.personalwealth.com>

Business week (McGraw-Hill site)

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Annual Returns 1926-2002

	USA			CANADA		
	S&P500	Treasuries	Excess	TSE	Canadas	Excess
Arithmetic	12.20	5.79	6.42	11.20	6.39	4.81
Geometric	10.20	5.40	4.80	9.56	6.03	3.53
Least squares	11.20	4.62	6.58	10.48	5.29	5.19
St Dev	20.49	9.41		18.80	9.07	

Arithmetic is simple average; geometric is compound and OLS is the least squares estimate of the average annual return.

Approximately Geometric Mean = Arithmetic Mean - .5*variance

For example, US variance is about 4%, so AM and GM diverge by about 2%

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US & Canadian Risk Premium

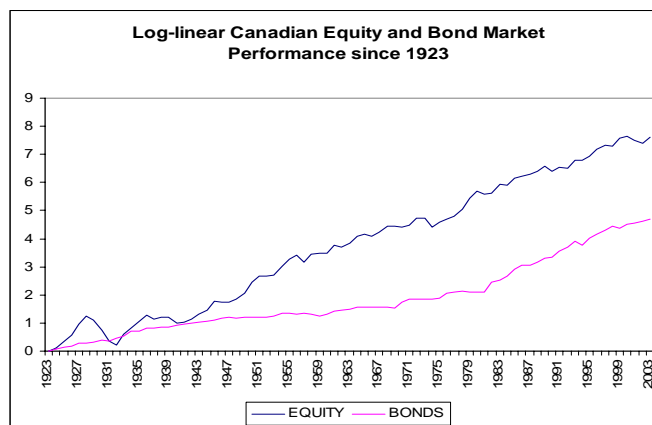
	Equity	Bonds	ERP
◆ Canada	11.20	6.39	4.81
◆ US	12.20	5.79	6.41
◆ Difference	+1.00	-0.60	1.60

*About two thirds one thirds equity bond markets
Standard error of the mean equity return is about 2.2%, so
difference isn't statistically significant*

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Log-linear Equity and Bond Market Performance

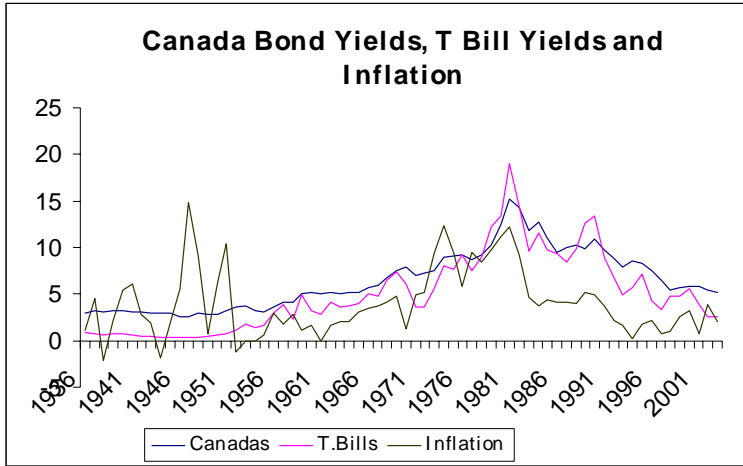


Slope is the continuously compounded rate of return

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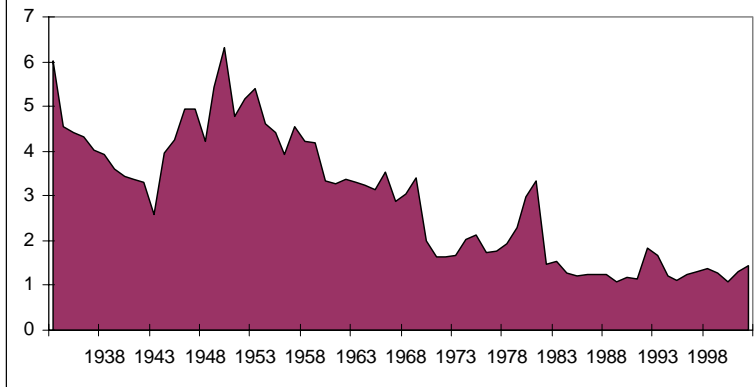
Bond Markets since 1936



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RELATIVE UNCERTAINTY Equities versus Bonds

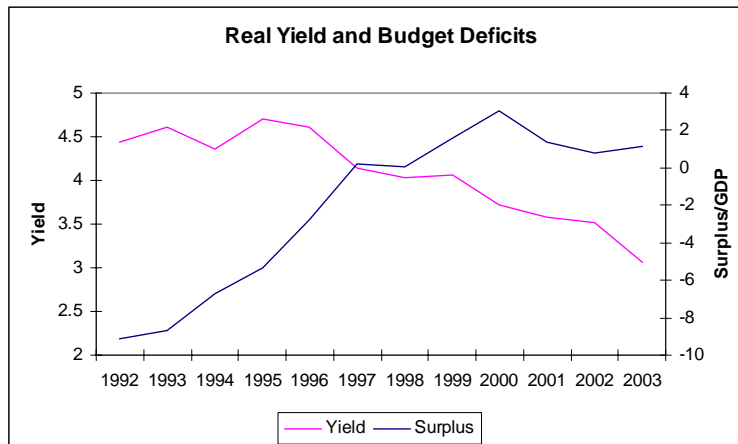


Ratio of 10-year rolling standard deviations of returns

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Real Bond Yields



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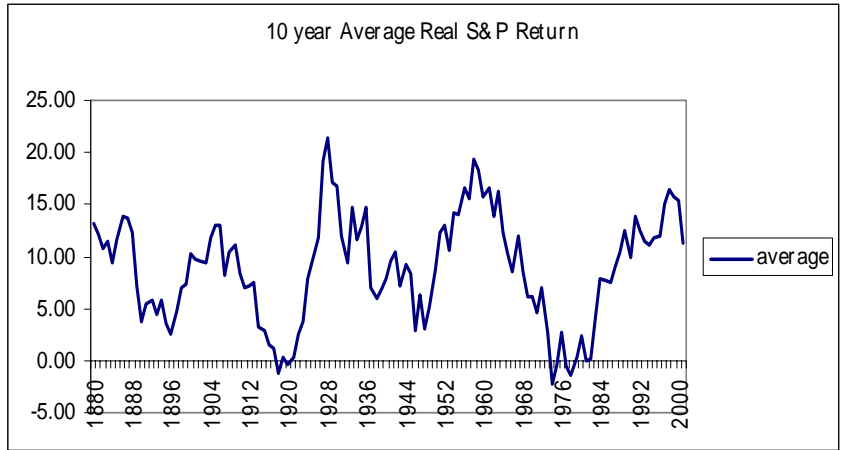


Conclusion

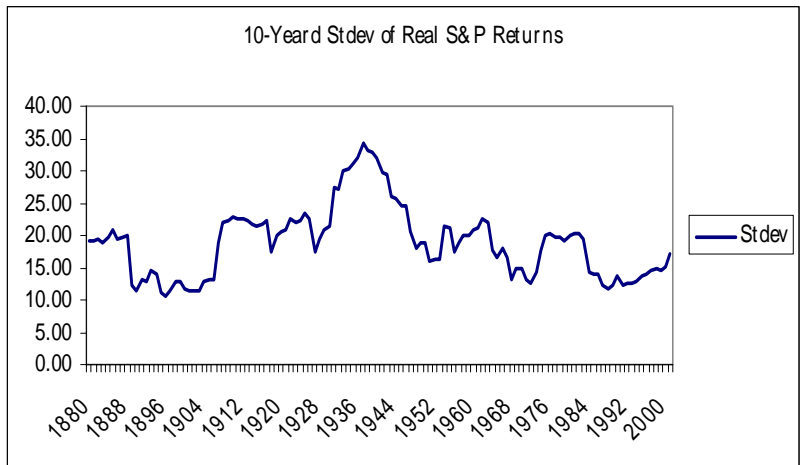
- ◆ **Equities outperform bonds**
 - About 5.0% in Canada
 - About 7.0% in US
- ◆ **Is Equity risk premium stable?**
 - Is equity return stable?
 - Bond return distinct periods
 - ❖ Up to early 1950's stable controlled interest rates
 - ❖ Up to 1981 rising interest rates
 - ❖ Since 1981 falling interest rates
- ◆ **Real bond yields have varied with the government deficit/surplus. Bond market risk depends on government**

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Implications

- ◆ **US equity market risk**
 - Decline from highs of 1920's-1930's
 - Before & after Great crash risk was lower
- ◆ **Arithmetic real equity return 8.8%**
- ◆ **Average standard deviation 18.5%**
- ◆ **Canadian returns about 1.0% lower**
- ◆ **Given a real bond yield of 3.6% this implies an equity risk premium of 5.3% for US and 4.4% for Canada**

- ◆ **What does this mean for portfolio risk?**

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Risk

- ◆ **Probability of incurring harm.**

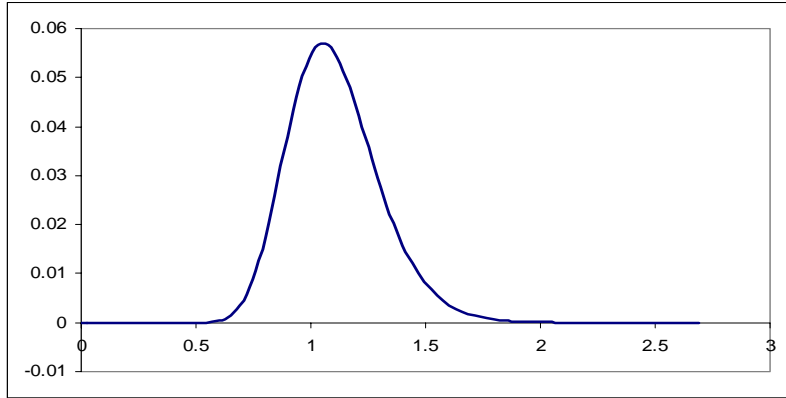
- ◆ **Harm is losing money or failing to beat the bond market**

- ◆ **The risk in the equity market is the likelihood of failing to beat a 3.6% real return even though you expect to earn 8.9%**

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One Period Investment



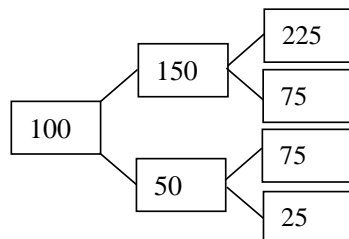
5.319% or 0.27 standard deviations below the mean or about 40% probability

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Multi-period

- ◆ Skewed distribution of multi-period returns
- ◆ Slim possibility of very high payoffs
- ◆ Suppose payoff is +/- 50%



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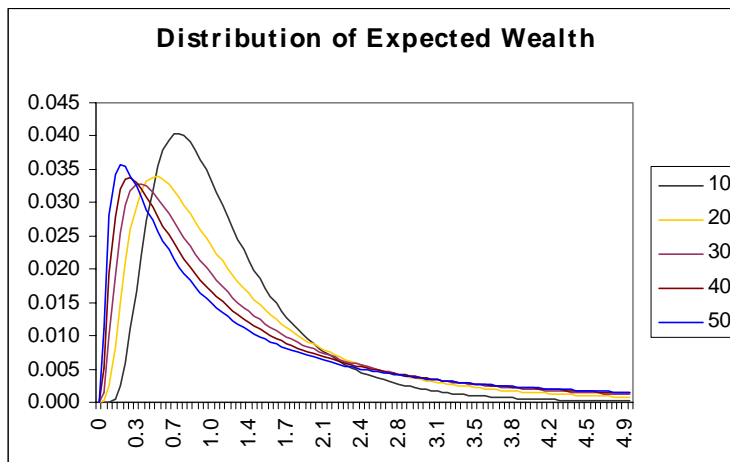


Multiplicative Random Variables

- ◆ **Central limit theorem**
 - Additive Random variables: approximately a normal distribution
 - Multiplicative random variables: approximately a lognormal distribution (the logarithm of the price is normally distributed)
- ◆ **Lognormal distribution**
 - Limiting distribution for all multi-period investments
 - Natural log of prices is normally distributed

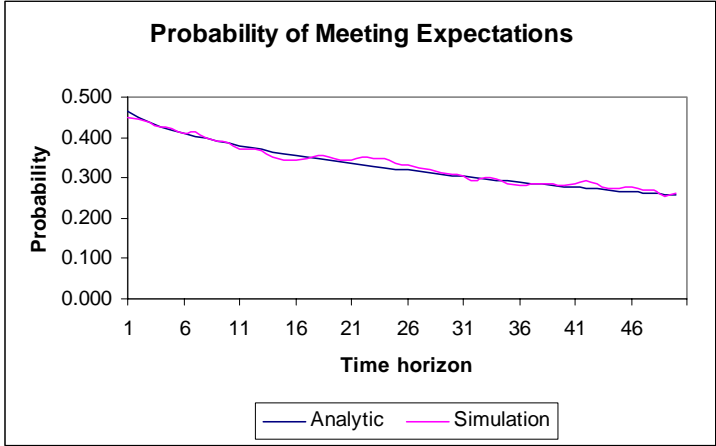
$$\text{Ln}(W_T) \sim \phi[\text{Ln}(W_0) + (r - .5\sigma^2)T, \sigma\sqrt{T}]$$

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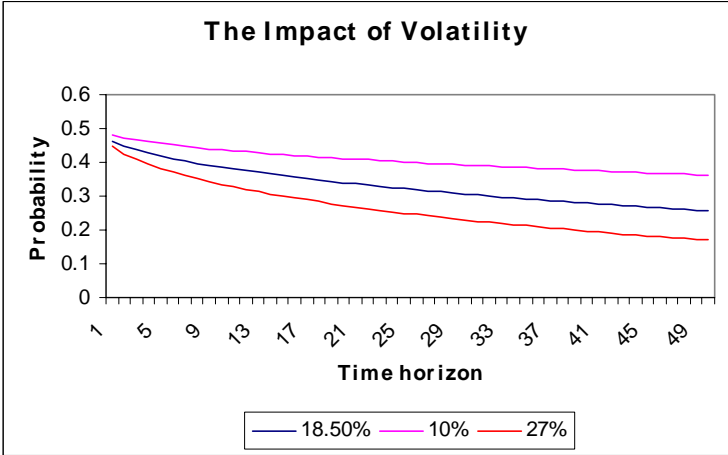


Planning on the basis of expectations is foolhardy!

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More Risk Means More Skewed Distribution



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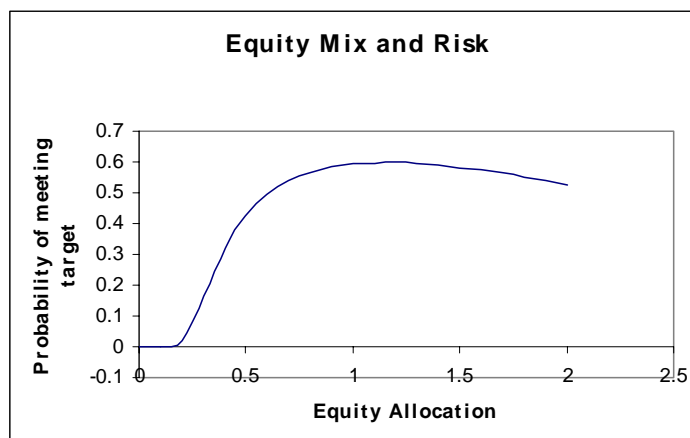
Retirement Problem

- ◆ Booth Financial Services Review (Spring 2004)
- ◆ Standard retirement target: “70 & 65”
- ◆ Retirement problem
 - Target of say 6.5%
 - Real bond gives 4.0%
 - Need equities
- ◆ More equities
 - Increase expected return
 - Increase skewed nature of return
 - Effect on probability of meeting target is ambiguous

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Retirement Problem on 50% equity Allocation “Expected” Basis



Higher equity allocations increased risk offsets increased expected return

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“Optimal” Equity Allocations

- ◆ Chance constraint

$$PR(Tr < Tk) < 30\%$$

$$N\left(\frac{k - E(r)}{\sigma\sqrt{T}}\right) < 30\%$$

- ◆ Minimise wealth level for target

$$\ln(W_*) = \ln(W_0) + T(k - (R + \alpha(r - R) - .5*\alpha^2\sigma^2) - .5244\alpha\sigma T^{-0.5})$$

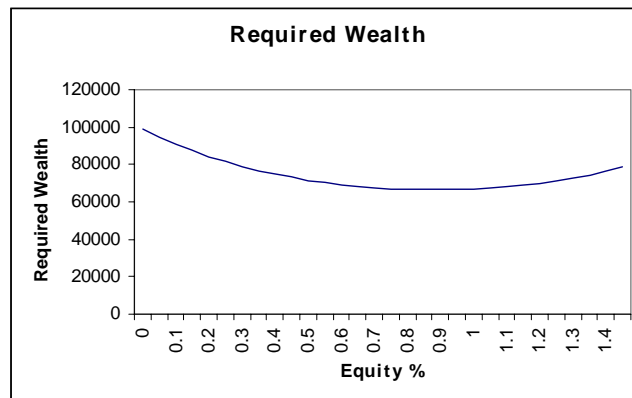
- ◆ Solution

$$\alpha = \frac{(r - R) - 0.5244\sigma T^{-0.5}}{\sigma^2}$$

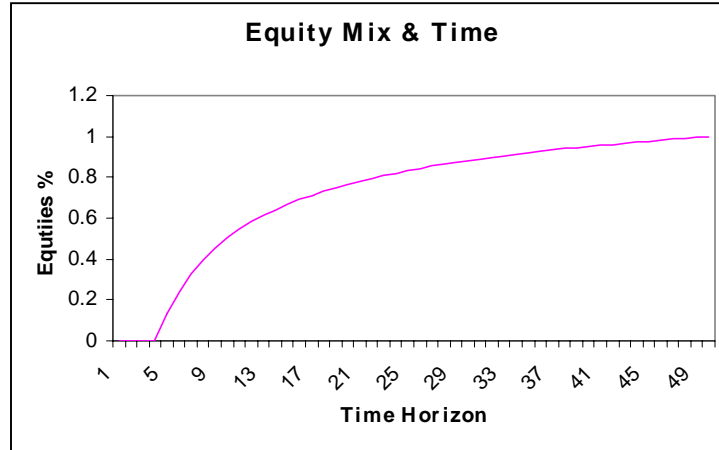


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Minimum Wealth



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Conclusions

- ◆ Not everyone should hold equities
- ◆ Equity allocation declines with investment horizon:
“Your age in bonds rule”
- ◆ Variation on non-EUT models
 - Objective function is log of terminal wealth
 - Chance constraint modifies the utility function for “downside” risk
 - Violates state independence assumption of EUT
- ◆ Tough convincing economists wedded to consumption-investment model and dynamic optimisation models

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