1 Drunk Gambler: Solution

Let k be the largest integer strictly less than $w_0/2$. That's the largest number of reds you can have without being ruined. Then

$$R_i = (1-p)\binom{i-1}{k}p^{i-1-k}(1-p)^k.$$

Let $j = i-1-k$, and $x = p(1-q)$. Then the relevant sum is

$$1 - \sum_{i-1\geq k}(1-q)^i(1-p)\binom{i-1}{k}p^{i-1-k}(1-p)^k$$
$$= 1-(1-q)^{k+1}(1-p)^{k+1}\sum_{j\geq 0}\binom{j+k}{k}x^j$$
$$= 1-\frac{1}{k!}(1-q)^{k+1}(1-p)^{k+1}$$
$$\times \sum_{j\geq 0}(j+k)\cdot(j+k-1)\cdots(j+1)x^j$$

But each term of that sum as the kth derivative of x^{j+k}. So the whole sum is the kth derivative of $\sum x^{j+k}$, which is the same thing as the kth derivative of $\sum x^{j}$. You can re-index the latter sum and note that the first batch of terms all vanish. That sum is just $1/(1-x)$, and its kth derivative is just $k!/(1-x)^{k+1}$.

Putting the bits together, the formula for Ruined While Sober (RwS) is

$$1-\left(\frac{(1-p)(1-q)}{1-p(1-q)}\right)^{k+1}.$$

Or, listed in the text:

$$1-\left(1-\frac{q}{1-p(1-q)}\right)^{k+1}.$$