



Dollar-Cost Averaging: Accumulation vs. Distribution

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Agenda..

- Examine the myth and legend of Dollar-Cost Averaging (DCA).
- Explain why DCA is suboptimal in the accumulation (pre retirement) phase.
- Argue that DCA is not much better in the distribution (post retirement) phase.
- Discuss two techniques for improving the odds of a successful retirement disbursement strategy.





When you invest regularly, you immediately have two advantages on your side.

1. First, your savings can continue to grow steadily through the magic of compounding. The benefits of compounding are significant when time is on your side. **As Albert Einstein is believed to have said, "Compounding interest is the greatest mathematical discovery of all time."**
2. Second, you can take advantage of dollar-cost-averaging to improve the efficiency of your investment. By purchasing a fixed dollar amount on a regular basis, you can purchase more units when the price is low, fewer units when the price is high and as a result, you can pay a lower average price for the units you buy.




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www.sharebuilder.com (dollar-cost averaging)



The Dictionary Says:


...Periodic investment of a fixed dollar amount, as in a particular stock or fund or in the market as a whole, on the belief that the average value of the investment will rise over time and that it is not possible to foresee the intermediate highs and lows...

 Invested	Price	# of Units	
Jan.	\$1,000	\$100.81	9.918
Feb.	\$1,000	\$115.40	8.665
Mar.	\$1,000	\$121.59	8.223
Apr.	\$1,000	\$125.99	7.936
May.	\$1,000	\$125.18	7.988
Jun.	\$1,000	\$124.25	8.047
Jul.	\$1,000	\$117.51	8.509
Aug.	\$1,000	\$125.97	7.937
Sep.	\$1,000	\$122.89	8.137
Oct.	\$1,000	\$136.71	7.314
Nov.	\$1,000	\$127.79	7.825
Dec.	\$1,000	\$121.02	8.263



Results....

- The end-of-year value of your DCA portfolio is \$11,952.90, which is 98.767 units times the final price of \$121.02.
- The arithmetic average stock/fund price during the year was \$122.10
- The average purchase price was \$121.50 (= total investment divided by number of units.)
- Average Cost < Average Price



Morningstar.com

“...The magic of dollar cost averaging is that you can end up with an average purchase price that is less than the average price over the time period. For example, when the price of albacore tuna fish averaged \$1.62 a can during the year, you paid an average purchase price of only \$1.56 a can...and you ended up with more cans of tuna for your money...”



Mathematically...

$$\frac{1}{N} \sum_{i=1}^N X_i \geq \frac{N}{\sum_{i=1}^N \frac{1}{X_i}}$$



The Alternative to DCA...

- If you have the money, invest right now in a well-diversified portfolio of stock/funds and fixed income products.
- Or, borrow against the total \$12,000 you will save next year, and invest a fraction in the stock/fund right now, and the remainder invest in the bank.

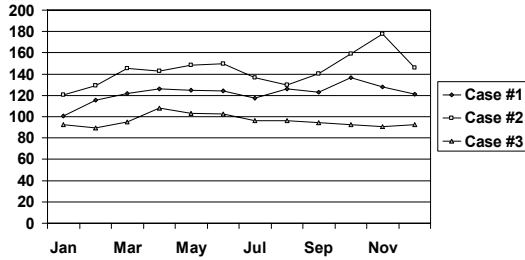


Lump Sum Investing (now):

Borrow Money at:	5%
Total Borrowed:	\$11,681
Allocate to Stock/Fund:	55%
Up-front investment of:	\$6,424.70
Remainder in GIC/Bonds:	\$5,256.55
End-of-Year L.S.I. Value:	\$13,300.60
End-of-Year D.C.A. Value:	\$11,952.90
L.S.I. Strategy Better By:	\$1,347.80



So, it all depends on the path...





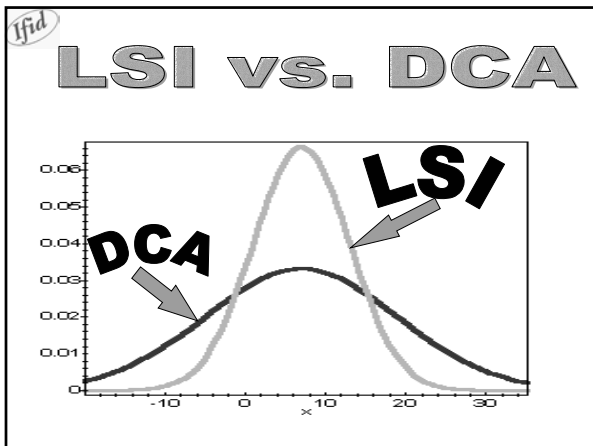
Is LSI better than DCA?

- We do not know how the stock market will perform during the next year. It might go up or it might go down. This is a random variable.
- Therefore, sometimes lump-sum investing will result in a better outcome than dollar-cost averaging, other times the opposite will occur.
- The proper question to ask is: What is the probability that LSI will outperform DCA?



Monte Carlo's Answer...

- We can answer these and many other stock market investing questions by using a computer to simulate thousands of different financial economic scenarios of the future.
- We can then compute the probability that one strategy is better than another.
- The odds do not look good for DCA!



Technical Results:

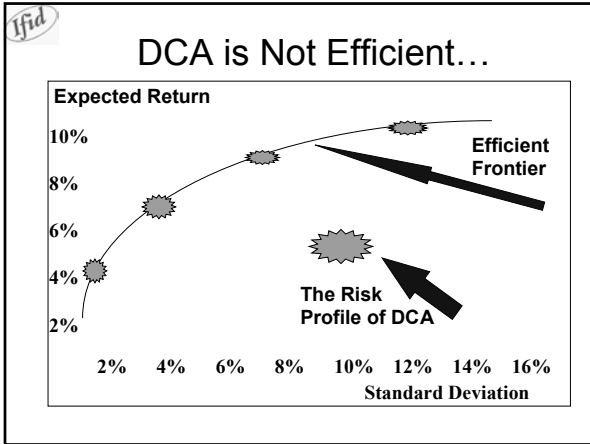
$$E[LSI] = E[DCA]$$

$$Var[LSI] < Var[DCA]$$

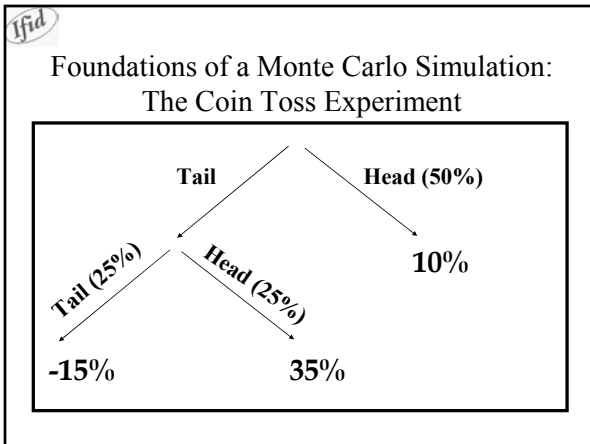
$$Pr[LSI \geq DCA] > 50\%$$

Dollar-Cost Averaging vs. Lump-Sum Investing

Equity Fund	Expected Wealth	Variability
\$10,000	\$11,000	\$1,500
\$5,660	\$10,783	\$850
\$4,960	\$10,748	\$744
\$0	\$10,500	\$0
DCA	\$10,748	\$850



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- Moving on to retirement...**
- A systematic withdrawal plan (SWiP) is the symmetric counterpart to DCA in the retirement phase.
 - Is (reverse) DCA optimal in the distribution phase?
 - No. It does not adjust for the wealth effect.
 - What are the odds that a SWiP will finance a pre-defined retirement lifestyle?





Long-term expectations.

- The Arithmetic Mean return from this investment is exactly 10%.
- Imagine you start with \$100 and stay invested for twenty years. What is your expected wealth at the end of the period?
- Answer $100 \times (1.1)^{20} = \672 , is wrong!
- True number is $100 \times (1.0855)^{20} = \516



Arithmetic vs. Geometric Mean

- The Arithmetic Mean of the coin-toss gamble is 10%; the Geometric Mean of the coin-toss gamble is only 8.55%
- The gap is often called the 'volatility drag'.
- The GM is a better indication of growth over the long term, compared to AM.
- Why? Imagine you earn -15% in 1st year, and 35% in the 2nd year. The total return is 14.75%, versus 21% from earning the Arithmetic Mean of 10% each year.



Greater volatility reduces the GM and the growth rate, even if AM remains unchanged.

	½	¼	¼	GM
	10%	35%	-15%	8.55%
	10%	40%	-20%	7.89%
	10%	45%	-25%	7.10%
	10%	50%	-30%	6.17%
	10%	60%	-40%	3.82%



Practical Implications: Financial Economic Forecasts

	Equity (Real)	Bonds (Real)
Arithmetic Mean:	7%	2.5%
Volatility: (30% Corr.)	20%	10%
Geometric Mean:	5%	2%



Lesson: The Impact of Skewness

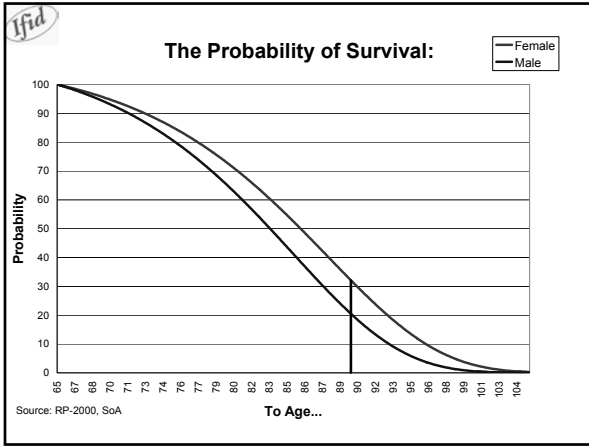
- The sequence of returns can be more important than the magnitude of returns.
- The greater the uncertainty the lower the growth rate, all else being equal
- Constant withdrawals have the same impact as highly skewed and negative returns.
- Immediate implications for Equity Premium calculations and model calibrations.



The Conditional Probability of Survival at Age 65

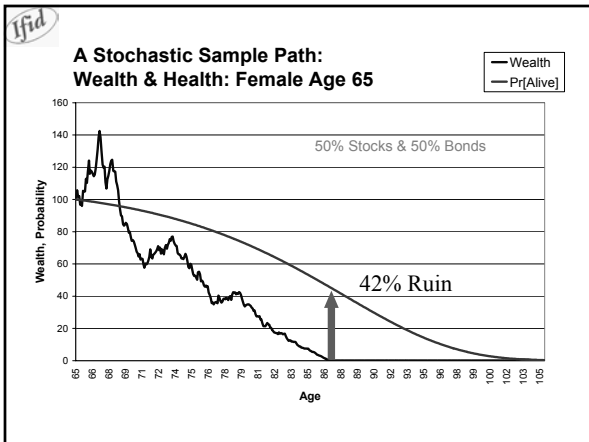
To Age:	Single Female	Single Male	At Least Member of a Couple
70	93.8%	92.0%	99.5%
75	84.4%	79.9%	96.9%
80	70.9%	62.7%	89.1%
85	52.8%	41.0%	72.2%
90	31.6%	19.6%	45.0%
95	13.4%	5.8%	18.4%

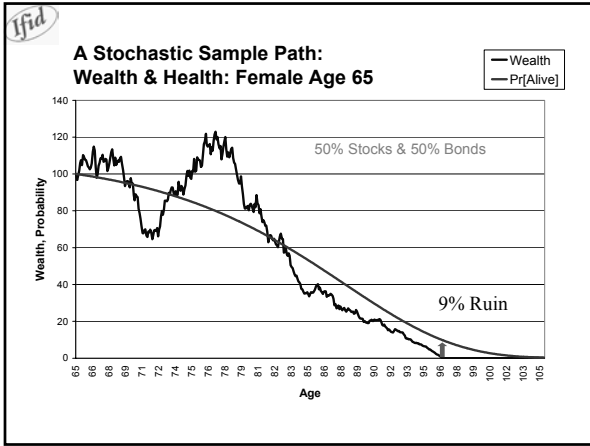
Source: Society of Actuaries RP-2000 Table

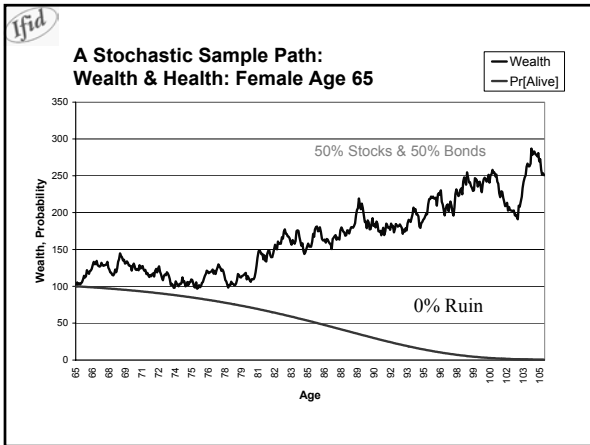


Probability of Retirement Ruin

- The probability the money runs out – assuming a fixed consumption strategy – prior to the date of death.
- It is the probability of dying after you are broke, as opposed to exactly when you are broke.
- Obviously, asset allocation has a strong impact on the Probability of Ruin.







**Probability of Retirement Ruin:
Equity vs. Bonds
Assuming Fixed Real Consumption**

Male 65	\$4 c.	\$6 c.	\$8 c.
0% Eq.	9%	37%	61%
25% Eq.	5%	28%	54%
50% Eq.	4%	22%	46%
75% Eq.	6%	22%	42%
100% Eq.	8%	24%	40%

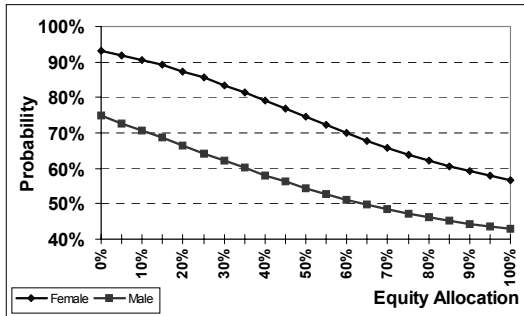


**Probability of Retirement Ruin:
Equity vs. Bonds
Assuming Fixed Real Consumption**

Female 65	\$4 c.	\$6 c.	\$8 c.
0% Eq.	14%	46%	69%
25% Eq.	8%	36%	62%
50% Eq.	6%	29%	54%
75% Eq.	8%	27%	49%
100% Eq.	11%	29%	46%



**The Probability of Retirement Ruin:
Age 65 with C = \$10**





**54% Ruin Probability
Who (and When) is to Blame?**

	Correlation of Return with Probability of Ruin
1 st Decade	-83%
2 nd Decade	-29%
3 rd Decade	-13%

50 % Eq. & \$8 real consumption



Can we reduce the ruin risk while maintaining the exact same asset mix?

- Yes: This is my main point.
- At retirement, we should be willing to trade some upside potential in exchange for downside protection.
- This will increase the median life-span of our income withdrawals.
- There is no free lunch, and financial insurance is paid by reduced future growth.



How does a 1-month collar work?

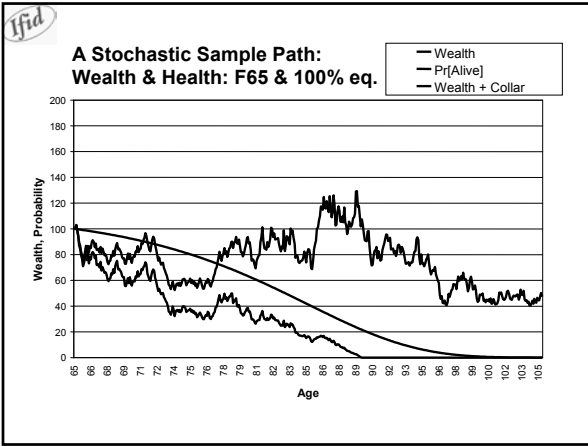
	Downside	Upside
Tight:	-5%	6.10%
Medium:	-7%	8.46%
Loose:	-10%	12.18%

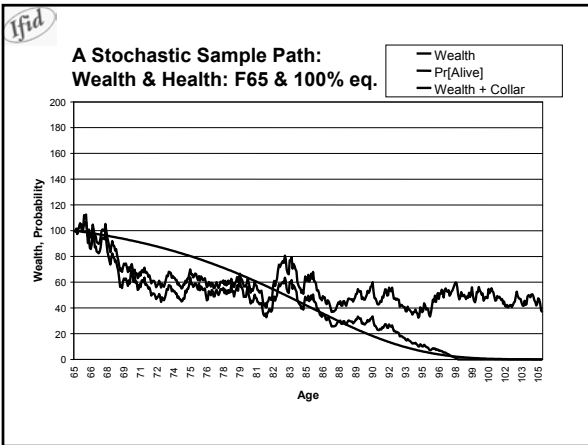
Assumptions: 30% implied volatility, 1.5% dividend yield.

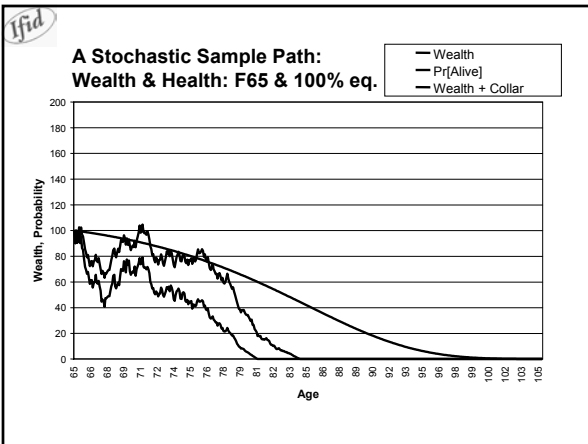


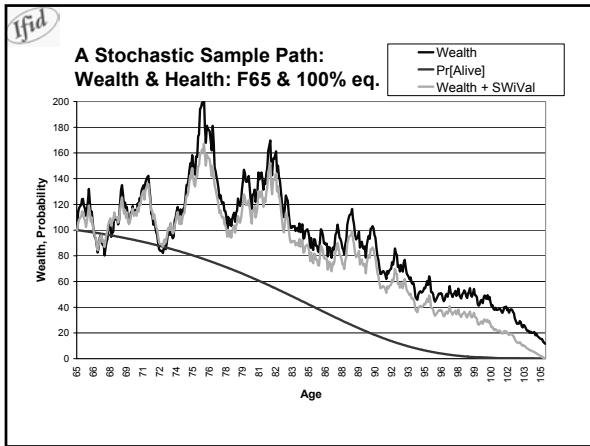
Introducing Retirement Collars

- Collar the investment return by trading upside against downside.
- This is similar to a strategy used to 'monetize' stocks and options granted to executives.
- We can generate MCS scenarios assuming various Collar parameters.
- For example: Assume 100% equity exposure and a 15 year collar of -5.0% against 6.10% per month.
- The same idea works with averaging returns.









Collared Probability of Ruin

Male 65	\$6 c.	\$5 c.	\$4 c.
100% Equity	24%	15%	8%
100% Equity + Collar	< 19%	< 10%	< 3%

Collared Probability of Ruin

Female 65	\$6 c.	\$5 c.	\$4 c.
100% Equity	29%	20%	11%
100% Equity + Collar	< 24%	< 13%	< 5%



Adding Longevity Insurance

- The probability of death for a 95 year-old is 20%
- Imagine five such females enter a joint agreement to invest \$100, in a 5% CD. Only those living to age 96 will share in the remains.
- On average, the pool of \$525 will be shared amongst four survivors.
- The \$131.25 represents a 31.25% return on investment, which is 26.25% more than the CD.
- The spread is called mortality credits, which form the basis of all annuity contracts.



Longevity Insurance = Life Annuities

- The mortality credits can be added to either a fixed or variable rate of return.
- Notice that mortality credits can subsidize investment losses, which might justify an increased exposure to financial risk.
- At any given age, the payout from a fixed life annuity is driven primarily by interest rates.
- There is a case to be made for 'slow' annuitization strategies that do not lock-in a long-term interest rate on a given retirement date.



Conclusion

- In the accumulation phase of retirement planning, Dollar-Cost Averaging is mean-variance inefficient and probably has more psychological value than economic value.
- In the distribution phase one can improve upon the odds of Retirement Ruin by using derivative securities and longevity insurance (i.e. annuitization strategies.)



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