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## **An Introduction to the Calculus of Retirement Income... ...for Poets**

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**Boomertirement Conference  
April 25, 2007**

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### The Next 60 minutes...

- Review recent **academic research** on retirement income planning.
- Focus on 3 Unique Retirement Risks.
- Introduce The Formula for Retirement **Ruin and Sustainability**.
- A Solution: The **FinSurance** products of the future.

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### Frozen Plans...

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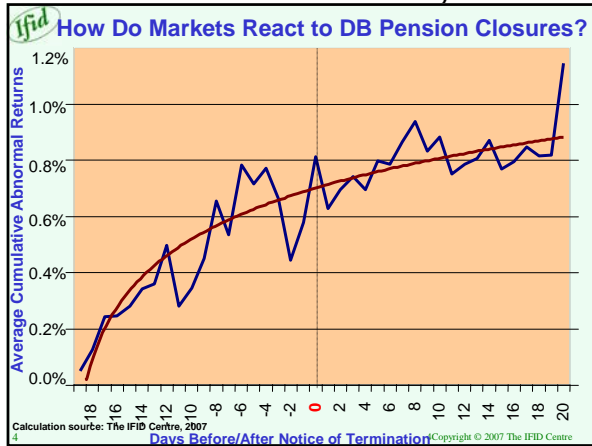
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**U.S. Retirees vs. Active Workers**

Company	Retirees per 100 Workers
Toyota:	2
Honda:	5
Nissan:	11
Ford:	125
Chrysler:	153
GM:	310

Source: New York Times, May 19, 2006 (C5)  
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You just got hired by one of the 100 largest companies in the U.S.  
**Did they offer you a D.B. Pension?**

Year	Year	Year
1985	2002	2005
<b>89</b>	<b>50</b>	<b>37</b>

Source: Watson Wyatt, Reported in J.F.P. Sep 2006  
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**What is Lost?**

- Pensions provide insurance against **longevity risk**; which is outliving your planning horizon.
- Most pensions include a cost of living adjustment which protects against **inflation risk**.
- DB formula and payout is immune to **sequence-of-returns risk**; the chance of early bear market.
- These **three risks** or hazards must now be managed by individuals, with your help.

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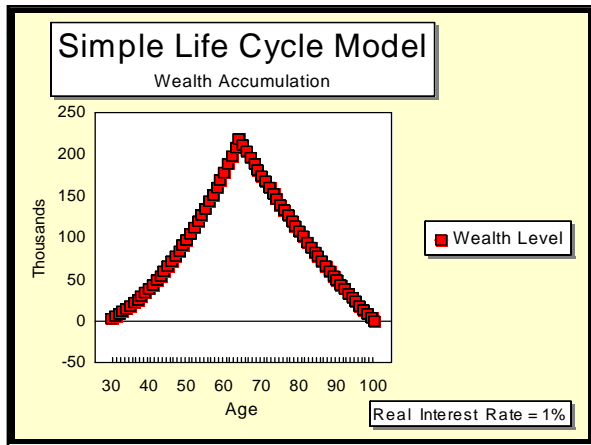
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**Exam Question for Today**

- You have a choice of two funds: **A** and **B**.
- Fund **A** is expected to earn 8% with a volatility (standard deviation) of 20%
- Fund **B** is expected to earn 7% with a volatility of 20%.
- They differ in no other material way.
- Which do you prefer?

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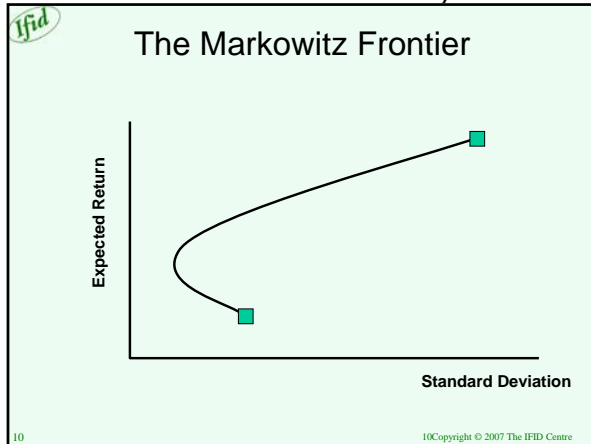
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**Retirement Income Planning & Sequence-of>Returns Risk**

Age 65 retirement wealth: \$100,000  
 Monthly withdrawal: \$750 (= \$9,000 per year)  
 Annual Investment return: Constant APR of 7%

**At what age will you run out of money?**

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**What stop did you get on the retirement merry-go-round?**

Return Sequence	Ruin Age	+/- Months
+7%, +7%, +7%...	86.50	14 Year Gap
+7%, -13%, +27%...	83.33	-38
+7%, +27%, -13%...	89.50	+36
-13%, +7%, +27%...	81.08	-65
+27%, +7%, -13%...	94.92	+101

\*Assumes \$9,000 spending per year.  
 12. Calculation source: The IFID Centre Copyright © 2007 The IFID Centre

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### The Retirement Income Formula (Alternative to Monte Carlo)

- Expected investment return (*AM*) and volatility risk (*VOL*) of your portfolio.
- Median remaining lifespan (*MRL*). This is the 50% point in the longevity tables.
- Inflation-adjusted spending rate (*S*), as a percent of initial retirement nest egg.

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### Retirement Alpha & Beta

$$\alpha = \frac{2AM + 2.773 / MRL}{VOL^2 + 0.6931 / MRL} - 1$$

$$\beta = \frac{2S}{VOL^2 + 0.6931 / MRL}$$

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### Retirement Ruin & Alpha Beta

$\alpha / \beta$	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75
4.00	0%	0%	1%	2%	4%	7%	10%	14%	19%	24%	30%
3.75	0%	0%	1%	3%	5%	9%	13%	18%	23%	29%	35%
3.50	0%	1%	2%	4%	7%	11%	16%	22%	28%	34%	40%
3.25	0%	1%	3%	6%	10%	15%	21%	27%	33%	40%	46%
3.00	0%	1%	4%	8%	13%	19%	26%	32%	39%	46%	52%
2.75	0%	2%	6%	11%	17%	24%	31%	38%	45%	52%	58%
2.50	1%	4%	9%	15%	22%	30%	38%	45%	52%	58%	64%
2.25	1%	6%	12%	20%	28%	37%	45%	52%	59%	65%	70%
2.00	3%	9%	17%	26%	36%	44%	52%	59%	66%	71%	76%
1.75	5%	14%	24%	34%	44%	52%	60%	67%	72%	77%	81%
1.50	8%	20%	32%	43%	52%	61%	68%	74%	79%	83%	86%
1.25	14%	28%	41%	53%	62%	69%	76%	81%	85%	88%	90%
1.00	22%	39%	53%	63%	71%	78%	83%	86%	89%	92%	94%
0.75	35%	53%	65%	74%	80%	85%	89%	91%	94%	95%	96%
0.50	52%	68%	78%	84%	89%	92%	94%	95%	97%	97%	98%

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**Numerical Example: #1**

- Expecting portfolio to earn  $AM=0.07$  real.
- Portfolio risk  $VOL=0.20$
- At retirement age 65 your  $MRL=25$  years.
- Initial spending rate is  $S=0.05$  of nest egg.



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**Numerical Example: #2**

- Expecting portfolio to earn  $AM=0.07$  real.
- Portfolio risk  $VOL=0.16$
- Retirement age 70 your  $MRL=21$  years.
- Initial spending rate is  $S=0.045$  of nest egg.



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**Homework Assignment:**

Spend	MRL	AM	VOL	Ruin
3%	18 yrs	9%	16%	?
6%	22 yrs	8%	15%	?
6%	28 yrs	7%	20%	?

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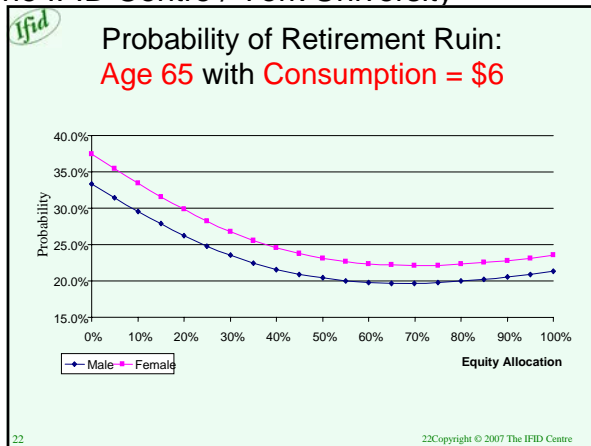
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### Important Takeaways...

- A balanced asset allocation improves the **sustainability** of the retirement income portfolio, even at advanced ages.
- There is a certain amount of **longevity & sequencing risk** that can not be removed with asset allocation alone.
- This is why we need **product allocation**.

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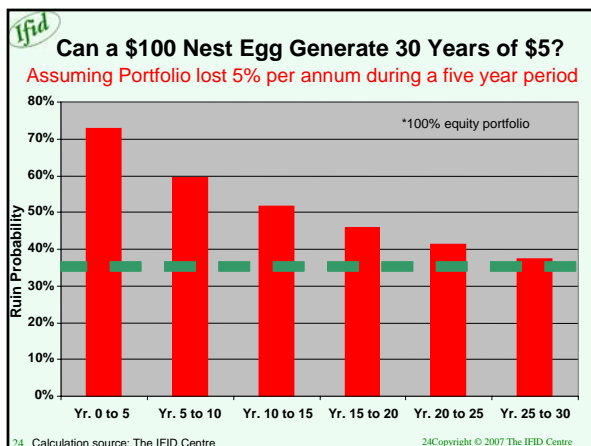
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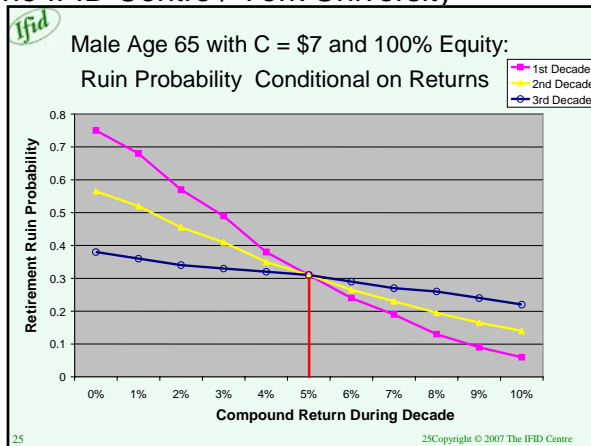
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**Inflation: What Does a \$1,000 Payment, Really Buy You?**

Year #	0%	1%	2%	4%
1	\$1,000	\$990	\$980	\$962
5	\$1,000	\$952	\$906	\$822
10	\$1,000	\$905	\$820	\$676
15	\$1,000	\$861	\$743	\$555
20	\$1,000	\$820	\$673	\$456
25	\$1,000	\$780	\$610	\$375
30	\$1,000	\$742	\$552	\$308
35	\$1,000	\$706	\$500	\$253

Source: The IFID Centre Calculations; the listed inflation rate was assumed to be effective and the calculations were based on continuous compounding of this rate

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**The elderly have a different CPI**

- Retirees face unique inflation compared to the rest of the population.
- The typical basket of consumption for a retiree differs from the one used to compute the CPI.
- This has substantial implications for retirement planning...

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**Consumer Expenditure Survey**  
U.S. Department of Labor (BLS) 2002

Annual Average	Age: 45-54	Age: 55-64	Age: 65-74	Age: 75+
Food & Alcohol	\$6,693	\$5,979	\$4,803	\$3,446
Medical Care	\$2,550	\$3,007	\$3,588	\$3,584
Multiple: F / M	2.62 x	1.99 x	1.34 x	0.99 x

Source: Journal of Financial Planning, June 2005, pg. 57 (Table #1)

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**Inflation: Relative Weights**

Category	CPI	CPI-E
Food:	17.9%	14.3%
Housing:	37.6%	45.9%
Apparel:	3.9%	2.8%
Transport:	18.8%	13.8%
Medical Care:	5.1%	10.2%
Recreation	5.3%	4.4%
Education	5.1%	2.9%
Other	6.1%	5.6%

Source: U.S. Department of Labor, Bureau of Labor Statistics (2004)  
Note: These weights change over time and are not fixed.

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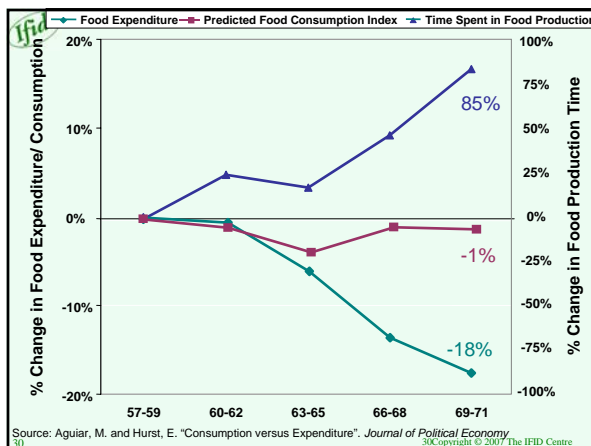
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 Ok, so how long should we plan for?

- **Wrong question!**
- **Correct approach:**
  - How do we manage the risk in planning for an unknown retirement horizon?

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 Life Expectancy: Birth vs. Age 75

	At Birth	At Age 75
<b>Male</b>	<b>73.6</b>	<b>84.6</b>
<b>Female</b>	<b>79.2</b>	<b>86.9</b>

Source: Life Tables for the United States Social Security Area 1900-2100 Actuarial Study No. 116.

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- *Best-practice female life expectancy...has risen during the last 160 years at a steady pace of almost **3 months per year**...*
- *One reasonable scenario would be that this trend will continue...life expectancy will reach **age 100 in about six decades**.*

**“Broken Limits to Life Expectancy”**  
**J. Oeppen and J.W. Vaupel**  
**SCIENCE, Vol. 296, 10 May 2002**

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### Health and Wealth: Life Expectancy at Age 70

Income Percentile	Healthy Male	Healthy Female
20 <sup>th</sup>	78.2 yrs	83.8 yrs
40 <sup>th</sup>	79.1 yrs	84.8 yrs
60 <sup>th</sup>	80.1 yrs	85.9 yrs
80 <sup>th</sup>	81.2 yrs	87.0 yrs

Source: Federal Reserve Bank of Chicago, WP 2005-13 (De Nardi, French and Jones)  
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### Probability of Survival at Age 65

To Age:	Female	Male
70	93.9%	92.2%
75	85.0%	81.3%
80	72.3%	65.9%
85	55.8%	45.5%
90	34.8%	23.7%
95	15.6%	7.7%
100	5.0%	1.4%

Source: Society of Actuaries RP-2000 Table with full projection.  
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### A Couple Aged 65....Survival:

Source: Society of Actuaries RP-2000 Table (with full projection)

To Age:	At Least One...
70	99.5%
75	97.2%
80	90.6%
85	75.9%
90	50.3%
95	22.1%

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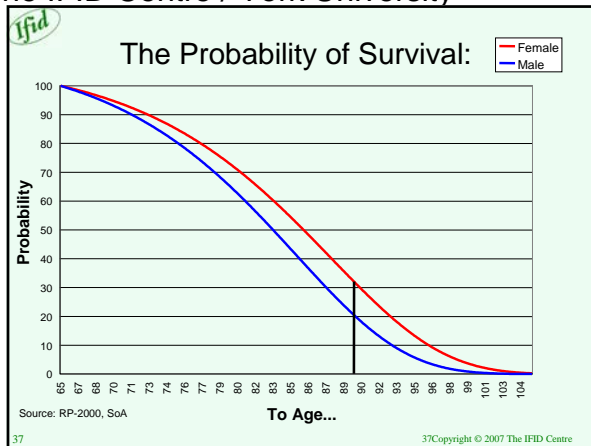
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	What will be cause of death?	
	Estimate	Actual
Heart Disease:	22%	34%
Cancer:	18%	23%
Other Natural:	33%	35%
Accident:	32%	5%
Homicide:	10%	1%
Other Unnatural:	11%	2%
<b>TOTAL:</b>	<b>126%</b>	<b>100%</b>

Source: LIMRA Survey 2004

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**N.B.E.R. Study**  
Based on Data from H.R.S.

- “...peoples’ perception of their own mortality risks are **systematically biased**...”
- “...many people respond with **0% and 100%** when asked to estimate the odds of living for **20** more years...”
- “...people with relatively **low life** expectancy tend to be **over-optimistic** and people with relatively **high life** expectancy tend to be **over-pessimistic**...”

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**Takeaway points...**

- Retirees face three distinct financial risks, which must be managed with your help & products
  - Longevity (mortality, health) Risk
  - Inflation (spending, consumption) Risk
  - Sequence-of>Returns (timing, market) Risk
- Longevity: Half your retired clients require at least a quarter of a century of income. Don't guess. Insure it.
- Inflation: CPI-E is personal. What asset keeps up?
- Sequence-of>Returns: A **7% average** can still create a **14 year** gap. Protect against your own bear.
- The main equation: **Ru + Su = 100%**

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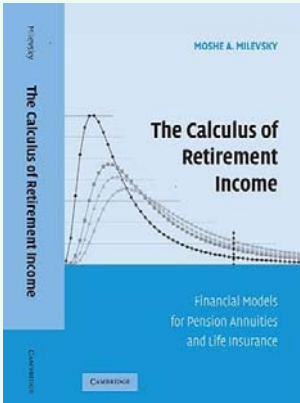
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