

Implied Life Credits: A Note on Developing an Index for Life Annuities

Moshe A. Milevsky¹

York University and The IFID Centre

Toronto, Ontario, Canada

20 December 2003

¹Moshe A. Milevsky is an associate professor of finance at the Schulich School of Business at York University and the Executive Director of the Individual Finance and Insurance Decisions (IFID) Centre at the Fields Institute in Toronto, Canada. He can be reached at milevsky@yorku.ca or at 416-348-9710 ext 3010. He would like to acknowledge Hua Zhang, George Gaspar and Jin Wang for research assistance, David Promislow for a number of helpful conversations as well as Lowell Aronoff (CANNEX) and Bruce Rhodes (PrecisionDatabases) for assistance with the data and the Society of Actuaries for partial funding.

Abstract

Implied Life Credits: A Note on Developing an Index for Life Annuities

In this paper the authors employ the concept of self-annuitization to develop a financial index for tracking the time series behavior of life annuity payouts. Aside from increasing awareness of the benefits of voluntary annuitization, there are a number of practical uses for such an index such as benchmarking relative competitiveness and analyzing the optimal age at which to annuitize.

From a technical point of view the proposed index goes beyond a (trivial) cross-sectional average of life annuity payouts offered by different insurance companies. Rather, the index is defined equal to the internal rate of return (IRR) that an individual would have to earn on their portfolio if they chose to self-annuitize, instead of purchasing a life annuity. We define this IRR – which is based on the current term structure of annuity payouts – as the *implied life credits* at a given age and for a given deferral period. The implied life credit value solves a non-linear equation that we proceed to approximate quadratically and for which we obtain a relatively simple and intuitive expression.

We then suggest age 65 – with a ten year period certain – compared against the same annuity at age 75, as the standard benchmark for the index. As an illustration of the concept we calibrate the index to a comprehensive time-series of weekly (Canadian) life annuity quotes for the years 2000 to 2003. During this period the implied life credits varied from 5.53% to 6.81% for males and 5.11% to 6.40% for females and is shown to be highly correlated with the yield on a 10-year Government of Canada bond.

JEL Classification: H55, G12, G22,

Keywords: Pensions, Financial Index, Insurance, Investments, Self-Annuitization

1 BACKGROUND AND MOTIVATION

There have been a number of recent papers in the pension and insurance literature that have explored the risk and return properties of a financial strategy known as self-annuitization during the retirement years. Self-annuitization is a consumption and investment strategy that attempts to closely mimic the payout from a life annuity while allocating assets to minimize the probability of lifetime ruin. This strategy might not be optimal within a classical life-cycle model with no bequest motives, as originally demonstrated by Yaari (1965) and recently extended by Brown, Davidoff and Diamond (2003). But the popularity and interest in this ‘annuity alternative’ continues to grow, especially given some of the imperfections in the life annuity market, as pointed out by Yagi and Nishigaki (1993).

In this paper the authors employ the concept of self-annuitization to develop a financial index for tracking the time series behavior of life annuity payouts. Aside from increasing awareness of the benefits of voluntary annuitization, there are a number of practical uses for this type of index such as benchmarking competitiveness and analyzing the optimal age at which to annuitize. From a technical point of view, the proposed index goes beyond a (trivial) cross-sectional average of life annuity payouts offered by different insurance companies. Rather, the index is defined equal to the internal rate of return (IRR) that an individual would have to earn on their portfolio, if they chose to self-annuitize, instead of purchasing a life annuity at a given age. We define this IRR – which is based on the current term structure of annuity payouts – as the *implied life credits* (ILC) at a given age and for a given deferral period. The (unique) implied life credit value solves a non-linear equation that is at the core of the paper. We approximate this equation to arrive at a relatively simple and intuitive expression for the ILC – which is the root of a quadratic equation – that can be easily computed on a pocket calculator.

We then suggest age 65 – with a ten year period certain – compared against the same annuity at age 75, as the standard benchmark for the index. This age range appears to be the most common one at which these decisions are made. As an illustration of the concept we calibrate the index formula and *implied life credits* to a comprehensive time-series of weekly (Canadian) life annuity quotes for the years 2000 to 2003. During this period, the implied life credits varied from 5.53% to 6.81% for males, and 5.11% to 6.40% for females, and is

shown to be highly correlated with the yield on a 10-year Government of Canada bond.

The remainder of the paper is organized as follows. Section 2 provides a basic numerical example to explain the mechanics of the index. Section 3 follows-up with the analytic representation. Section 4 provides an examination of the historical annuity data using the ILC index. Section 5 provides some additional insights and derives an easy to use approximation, while Section 6 concludes the paper.

2 THE ANNUITY INDEX: EXAMPLE

On November 26, 2003 a 65 year-old Canadian male would have been able to convert a \$100,000 lump-sum (tax sheltered) premium into a life annuity by going to any one of ten or so insurance companies that offer competitive quotes on life annuities. According to data compiled by CANNEX Financial Exchanges and The IFID Centre, these companies would have quoted him a payout ranging from a high of \$690 per month (Empire Life) to a low of \$633 per month (Great West Life). These numbers assumed he was interested in acquiring 10 years of guaranteed payments, and that the remaining payments would continue as long as he lived. If he wanted a longer guarantee period, or perhaps payments to go to a spouse in the event of his death, the monthly payout would be lower. In contrast, if he was willing to settle for a lower guarantee period, he would receive more income per month. But, according to most insurance companies, the 10 years of guaranteed payment is a common request amongst retirees.

Recall that an annuity with a ten-year (payment certain) guarantee can be broken into two components. The guaranteed portion is similar to a coupon-bearing bond. The other portion makes payments to the annuitant after the end of the payment certain period, only if the annuitant survives the guaranteed period. Implied in the pricing of the life annuity is a probability that the annuitant will die within the ten year guaranteed period. Should this occur, the annuitant's estate would not receive payments after the guarantee period is over. Insurance companies pool risk and – as a direct result of the possibility the individual will not receive a full return of their original payment – the actual payments will be higher to annuitants that survive the guarantee period. The Implied Life Credits index we are proposing measures how much higher those payments would be.

FIGURE #1 HERE

Figure #1 displays the payouts (per month) for all the insurance companies quoting such annuities on a weekly basis during the last three years. Notice the wide dispersion of up to 15% between the highest and lowest companies. Part of this can be attributed to the credit rating of the company (higher rated companies pay less) and part can be attributed to the general appetite of the company for taking on more annuity business. For example, Empire Life tends to show-up at the top of most income comparisons, but the company is ranked a solitary "A" by A.M. Best. In contrast, Great West Life appears on the bottom, but has a coveted "A++" credit rating. The credit/income relationship is quite robust. Nevertheless, if we (arbitrarily) take the average of the five highest annuity payouts quoted to a 65-year-old male with a \$100,000 premium, we get approximately \$678.22 per month. On November 26th, 2003 this consisted of Empire Life (\$690), Maritime Life (\$679), Desjardins (\$679), Equitable Life (\$672) and Transamerica Life (\$669). The \$678.216 number will form the basis of our index on November 26th, 2003.

On the same date, a 75 year-old male would have been able to convert a \$100,000 premium into a much higher monthly payment ranging from \$1,002 per month (Empire Life) to \$948 per month (Sun Life). In this case, the average of the five best quotes was \$975.904 per month. Stated differently - and this is the key to the implied life credits index - if a 75 year-old male wanted to purchase a life annuity with a zero-year guarantee paying the original \$678.22 per month, he would only have to pay $(\$678/\$977)*\$100,000 = \$69,396$ or roughly 70% of the original cost. The same annuity would be cheaper, if purchased later. A 65 year-old requires a \$100,000 premium to generate \$678 for life (with 10-years of certain payments), while a 75 year-old requires only \$69,396.

What would happen if the 65 year-old male decided to forgo the purchase of a life annuity and instead invested the \$100,000 in a well-balanced (low cost) mutual fund, and then withdrew the same exact \$678.216 per month for the next 10 years? This strategy is called self-annuitization and has been explored by a large number of recent papers in the pension and insurance literature. What would be the required portfolio investment return needed to successfully withdraw \$678.216 per month AND still have \$69,396 at the end of ten years to purchase an identical annuity?

This number is precisely what we call the Implied Life Credits at age 65. In the above

example, the number works out to 5.90%. We will demonstrate precisely how to compute this number in the subsequent section. But, if the 65 year-old can earn a compound annual return of 5.90%, he will be able to purchase (in expectation) the exact same life annuity at age 75 as he could have at age 65. If we go through the same exact calculation for a female, it would approximate to 5.46%. As a means of comparison, the ILC values can be compared to the 10-year Government of Canada Bond yield. On November 26th the bond yield was 4.79%. The ILC value for males (females) was approximately 111 (67) basis points above the bond yield¹. On the same date the average yield on a long-term high-quality corporate bond (proxied by the Scotia Capital AA bond index) was 6.27%. The yield on a corporate bond exceeded the ILC on the annuity. Stated differently, if the retiree could lock-in the 6.27% yield to maturity and at the same time lock-in a forward price for the life annuity at age 75, they could stochastically dominate the pay-off from purchasing the annuity at age 65.

How can this number be used? There are several important uses to such a metric, and therefore, good reasons for it to be computed and reported on an ongoing basis. The ILC should help consumers understand (and decompose) exactly what they are getting when they purchase a life annuity. In fact, one can obtain ILC values using the same algorithm to compare any two ages. Thus, one might compute the ILC for someone aged 70 or 75 who is contemplating purchasing a life annuity versus waiting to age 80 or 85. In the same manner, consumers can compute the ILC from a Defined Benefit pension at any age.

As a general fact, the older the age group, the higher these implied life credits become. In fact, in the mid 80s these numbers can get quite high, which implies that the insurance + financial return from these instruments far exceeds the investment return available from a Government Bond. Or, vice versa, it will be very hard to beat the returns from life annuity using any other financial instrument. The next section will derive the analytics.

¹Please note that the bond yield was not actually computed from market prices. Rather, the yield was taken from the Bank of Canada website which reports these and other official statistics on an ongoing basis. Quite likely they make a number of approximations with regards to the precise maturity, treatment accrued semi-annual coupons as well as the bid-ask spread. Therefore, the 4.79% – which is not necessarily the focus of our paper – should be taken as a rough approximation to the true yield in a strict mathematical sense.

3 INDEX ANALYTICS

With some abuse of actuarial notation we start by letting a_x^τ denote the price of a fixed immediate annuity (FIA) that pays \$1 per annum for life – but with τ years of certain payments – when the individual is currently at age x . In our model, the pure actuarial annuity factor will satisfy:

$$a_x^\tau = (1 + L_x) \left(\int_0^\tau e^{-(r_t)t} dt + \int_\tau^\infty ({}_t p_x) e^{-(r_t)t} dt \right), \quad (1)$$

where $({}_t p_x)$ denotes the conditional survival probability, L_x denotes the (possibly age dependent) loading factor and r_t denotes the (net) spot-rate curve used to discount cash-flows. When $\tau = 0$ which implies a zero payment certain, the annuity factor will be a_x and the superscript will be abandoned. The collection a_y for $y = 0 \dots 120$ represents the term structure of annuity factors on any given date. Note that equation (1) is fully consistent with a financial economic No-Arbitrage pricing of life annuities, as demonstrated by Carriere (1999). And, while this paper does not require an actual parameterization of the mortality law in continuous time – an example would be the Gompertz Makeham law whose properties have been promoted by Carriere (1994) or Frees, Carriere and Valdez (1996) – we can assume this curve exists without any loss of generality.

The theoretical basis of our Implied Life Credits (ILC) index is to compute the *internal rate of return* that an x -year old would have to earn on their non-annuitized portfolio over the next τ years in order to replicate the income payout from the annuity, and still be able to acquire the same income pattern at age $x + \tau$. The concept underlying self-annuitization was originally introduced by Khorasane (1996) and Milevsky (1998) and subsequently investigated by Kapur and Orszag (1999), Milevsky and Robinson (2000), Albrecht and Maurer (2002), Blake, Cairns and Dowd (2003), Gerrard, Haberman and Vigna (2003) as well as recent working paper by Dushi and Webb (2003), Young (2003) and a practitioner-oriented paper by Reichenstein (2003)

To understand the analytics of self-annuitization, we begin with a hypothetical retiree who has $W_0 = w$ dollars in marketable wealth. If this individual were to annuitize – i.e. to convert a stock of wealth w into a lifetime flow – he or she would be entitled to w/a_x^τ per annum for life. If, in contrast, the retiree decided to forgo the purchase of the life annuity and instead decided to self-annuitize – by investing the funds at a force of interest denoted by

δ and consuming at the annuity rate w/a_x – the wealth dynamics would satisfy the Ordinary Differential Equation (ODE):

$$dW_t = \left(\delta W_t - \frac{w}{a_x^\tau} \right) dt. \quad (2)$$

In words, the instantaneous change in the value of the portfolio would be the sum of the interest gain (δW_t) minus the withdrawal for consumption purposes (w/a_x^τ). Note, of course, that δ is assumed constant (non-stochastic) over time.

The solution to the Ordinary Differential Equation (ODE) in equation (2) is:

$$W_t = \left(w - \frac{w}{\delta a_x^\tau} \right) e^{\delta t} + \frac{w}{\delta a_x^\tau}, \quad (3)$$

as long as W_t is positive and δ can always be selected so that $W_t > 0$ for all values of t . But, if this investment portfolio is to contain enough funds to purchase the *same exact annuity flow* at age $x + \tau$, the following relationship must hold:

$$\frac{w}{a_x^\tau} a_{x+\tau} = \left(w - \frac{w}{\delta a_x^\tau} \right) e^{\delta \tau} + \frac{w}{\delta a_x^\tau}. \quad (4)$$

The intuition behind equation (4) is as follows. The right-hand side describes the evolution of wealth under a consumption rate of (w/a_x^τ) and an interest force δ . The annuity factor $a_{x+\tau}$ represents the cost of acquiring a dollar-for-life at age $x + \tau$. Thus, the cost of acquiring the original life annuity flow (w/a_x^τ) at age $x + \tau$, is exactly the left-hand side of $(w/a_x^\tau)a_{x+\tau}$. We are then searching for a value of δ that equates both sides. If δ is ‘too small’ then the left-hand side will be ‘too expensive.’ In contrast, if δ is ‘too large’ then the individual can afford a better annuity.

Dividing by w and multiplying by a_x^τ , we are left with:

$$a_{x+\tau} - \left(a_x^\tau - \frac{1}{\delta} \right) e^{\delta \tau} - \frac{1}{\delta} = 0 \quad (5)$$

The value of δ^* that solves the above equation will be the implied life credits. It is the rate that must be earned on non-annuitized wealth to be as well-off after τ years.

We demonstrate equation (5) with the help of the numerical example we presented in the previous section. On November 26th, 2003, a 65-year-old male is quoted an average monthly payout of \$678.216 per initial premium of \$100,000 with a 10-year payment certain period. The continuous-time annuity factor is approximated as $100000/(12 * 678.216) = 12.2871$ which is $a_{65}^{10} = 12.2871$ per \$1-for-life using our notation. On the same exact date, a

75-year-old is quoted an average monthly payout of \$977 per premium of \$100,000 with a zero-year payment certain period. The annuity factor is $100000/(12 * 975.904) = 8.5391$ which is $a_{75} = 8.5391$ per \$1-for-life².

We are searching for the implied rate of return δ that the 65-year-old would have to earn on their discretionary investment portfolio to beat the return from the annuity, but still consume the exact same income on an ongoing basis. The situation we are faced with is equation (5) with $\tau = 10$ years, $x = 65$ and δ being the unknown return variable.

$$8.5391 - \left(12.2871 - \frac{1}{\delta}\right) e^{10\delta} - \frac{1}{\delta} = 0 \quad (6)$$

The precise solution, which must be computed numerically due to the non-linearity of the equation, is $\delta^* = 0.0590$ which is an ILC value of 5.90%. As stated earlier, the 65-year-old male would have to earn 5.90% per annum each year for the next 10 years to beat the return from the annuity. Ergo, the value of the ILC index on November 26th, 2003 is 5.90% for male. The same calculation can be done for females using the average payouts listed earlier. In this case, $a_{65}^{10} = 13.3706$ and $a_{75} = 9.7875$ for a value of $\delta^* = 5.465\%$. Naturally, the δ^* value is lower since the (expected) horizon over which the payments are being returned is longer.

On a technical level, we solve for the unknown δ value using numerical techniques by taking the left hand side (LHS) of equation (5) and treating it as a function $f(\delta)$. The δ we are searching for is the root of $f(\delta) = 0$. We then use a Newton-Raphson (NeRa) algorithm to locate the δ , the root of the equation $f(\delta)$. The NeRa algorithm is based on Taylor expansion of the function $f(x)$ in the neighborhood of a point x :

$$f(x + \varepsilon) \approx f(x) + f'(x)\varepsilon + \frac{f''}{2}\varepsilon^2 + \dots \quad (7)$$

For small enough values of ε , the terms beyond $f'(x)\varepsilon$ are of second-order importance, hence $f(x + \varepsilon) = 0$ implies:

$$\varepsilon = \frac{-f(x)}{f'(x)}. \quad (8)$$

²We are fully cognizant that our formula requires a continuously paying annuity and that the data we are working with only include monthly pay annuities. For the sake of transparency, we decided to forgo some of the actuarial approximations that are commonly used to convert monthly into continuous annuity factors given that we lack a formal interest rate – which is a critical input to such approximations – and more importantly, because it makes a minimal difference to the final delta values.

Thus, when we are trying to locate a value of δ such that $f(\delta) = 0$, we start with an initial $\delta = \delta_0$, and then by the NeRa algorithm we pick the next value of δ so that:

$$\delta_{i+1} = \delta_i - \frac{f(\delta_i)}{f'(\delta_i)}. \quad (9)$$

We then follow this process until $|\delta_{i+1} - \delta_i| < \varepsilon$, where ε is a small-enough value, which in our case is three significant digits after the decimal point. In a later section we will provide a relatively accurate quadratic approximation for δ^* that yields some additional insight into the structure of the implied life credits.

4 DATA CALIBRATION

This section uses data provided by CANNEX Financial Exchanges and The IFID Centre – based in Toronto, Canada – to calibrate the index we described in the previous section. In terms of background, CANNEX compiles ongoing quotes from, what we believe to be, *most* insurance companies in Canada that market and sell life annuities. On any given day, a (subscribed) user can log-into CANNEX’s secure website and query their system for a list of available quotes on a given annuity type. Their system then displays all companies that are offering to sell that particular product type (i.e. age, gender, joint-life, payment certain, premium size, etc) and the user can then perform their own comparison of these quotes and then contact the ‘best’ company directly to actually purchase the annuity contract. CANNEX is not a broker *per se*, but an intermediary that compiles quotes and charges a fee for this information. The user then contacts the insurance company with the (best) desired quote directly.

To get a sense of their influence on the market, in the year 2002 alone, subscribed users requested over 46,000 paid ‘queries’. We believe that the existence of CANNEX is unique on a global scale in the ability to comprehensively survey the entire national market for the best quote at any given time. This commoditization of life annuities was initially discouraged by a number of Canadian insurance companies, but eventually CANNEX prevailed and, with the exception of Industrial Alliance Insurance Company, they all currently participate in the quotation system. To provide a further sense of the magnitude of the (payout) life annuity market in Canada, according to the industry publication *The Insurance Journal*, in the first

quarter of 2003 over \$542.5 million (\$1 CAD = \$0.77 USD) of single premium immediate annuities were sold in Canada. This represented a growth rate of 68% over the sales during the first quarter of 2002.

Notably, CANNEX does not actually store any of these quotes in an accessible time series and historical format. CANNEX has entered into an agreement with The IFID Centre (a non-profit organization based at the Fields Institute for Research in Mathematical Science) to gather, compile and organize these quotes for a number of representative product types so that a permanent record can be maintained of their evolution over time. The IFID Centre's annuity database captures annuity quotes for ages 55, 60, 65, 70, 75 and 80 for single males, females and a variety of joint-life scenarios and guarantee periods. This database has been operational for the last four years and the authors have used these numbers as the primary source for this article. Once again, Figure #1 displays a sample of these quotes for all companies.

There are a number of interesting observations that are evident from Figure #1. First, there has been a noticeable downward trend in payouts (i.e. increase in annuity factors) at age 65 over the last three years. This is likely driven by the reduction in medium to long-term interest rates over the period in question. More interestingly, note that payouts can vary quite significantly from company to company, and certain companies are consistently (un)competitive in this market. As a comparison, Table #1 provides some summary statistics for yields on 10-year Government of Canada bonds over the time period in question.

TABLE #1 HERE

The next step was to average the five best annuity quotes for males and females at age 65. These form the input for the ILC index we have constructed. As mentioned earlier, the one or two highest quotes tended to be from insurance companies with relatively lower credit ratings. Of the companies that were consistently in the top five – namely Empire Life, Equitable Life, Maritime Life, Transamerica Life and Canada Life – 3 are rated single A or lower by A.M. Best.

For each and every Wednesday over the period October 2000 to October 2003 we computed the a_{65}^{10} and a_{75} annuity for both males and females by dividing the \$100,000 into 12 times the monthly payout. We thus generated a series of 160 male and female pairs

$\{a_{65}^{10}(i), a_{75}(i); i = 1..160\}$ which were then inserted into equation (5) to then solve for the relevant δ^* on that date. According to Table #1, the average ILC value during the 2000 to 2003 period was 6.374% for males and 5.956% for females. The gap between males and females of approximately 40 basis points is consistent during the entire period. Males must earn a higher benchmark return to successfully ‘beat’ the return from a life annuity. This hurdle rate obviously increases with age.

FIGURE #2 HERE

Figure #2 illustrates the evolution of the ILC values during the three-year period, compared to the yield on a 10-year Government of Canada Bond. We have chosen the yield on this particular fixed-income instrument given its centrality in many of the insurance companies pricing algorithms as well as the fact that it is a reasonable investment alternative to actually purchasing a life annuity at age 65. An interesting point of note is the sharp decline and then increase in the bond yield around the September 11th, 2001 period. Annuity quotes (per \$100,000) declined as well – as evidenced by Figure #1 – and the implied life credits declined, but quickly recovered within two or three weeks of the terrorist attacks.

FIGURE #3 HERE

Figure #3 goes one step further and subtracts the ILC values from the yield on a 10-year Government of Canada Bond. Note the mean reverting nature of these numbers around the 114 basis points for males and 72 basis point for females. In sum, the ILC index is relatively stable over short periods of time, is highly correlated with prevailing interest rates and is relatively easy to derive and explain to the non-specialist.

5 ADVANCED ISSUES

There are a number of corollary issues that are raised by the above methodology, as well as some *ad hoc* assumptions we have made along the way.

5.1 Additional Age Points.

First, there are a large number of possible Implied Life Credit (ILC) values that can be computed. All that is needed are two distinct annuity age/quotes and equation (5) provides

a delta value. In fact, the Implied Life Credits (ILCs) should properly be indexed by both age as well as the implicit delay period. Indeed, there is nothing special about δ_{65}^{10} other than it represents one of the most popular life annuities purchased (and quoted) in practice. For example, on the same November 26th date, the payout per \$100,000 premium for a joint-and-last-survivor annuity with a 10-year payment certain would be \$571 per month if purchased from Maritime Life (which was the best company for that particular quote). At age 75 a couple could obtain \$727 per month for life on a joint-and-last-survivor, but with zero payment certain. Putting these numbers thru equation (5) leads to a $\delta^* = 5.21\%$ which is a mere 38 basis points above the yield on the risk-free 10-year Government of Canada bond and much lower than the 6.27% yield on long-term corporate bonds. Once again, this illustrates the versatility of the ILC concept in explaining the relatively ‘low’ longevity insurance value embedded within a life annuity when full guarantees and survivor periods are imposed. Along the same lines – but perhaps in the other direction – a 75 year-old male with \$100,000 would have be secured \$958.50 per month (best quote from Empire Life) with a 5 year payment certain. An 80 year-old male would have obtained \$1,1234.29 per month (best quote from Maritime Life) assuming a zero payment certain. This works out to $a_{75}^5 = 8.6941$ and $a_{80} = 6.7515$, for an ILC value of $\delta^* = 10.281\%$. The 75 year-old would have to earn a 10.281% return over the next 5 years to ‘beat’ the life annuity. This far exceeds the 5.90% value listed above (at age 65) and appropriately illustrates the power of longevity insurance. We believe that these types of illustrations – comparing the ILC values of age x against age y – are one of the main benefits of creating and maintaining such an index.

5.2 What Moves Delta?

If the typical insurance company priced life annuities based solely on the constant yield of the 10-year Government Bond – and did not use a full term structure model – deviations in δ_{65}^{10} over relatively short-periods of time would be fully explained by changes in the bond yield. To test whether this is the case, we regressed changes in the implied life credits – i.e. the quantity $\delta(i) - \delta(i - 1)$ using abbreviated notation – on changes in the bond yields $y(i)$ over time. The exact specification we tested was:

$$\ln \left[\frac{\delta(i)}{\delta(i - 1)} \right] = c_0 + c_1 \ln \left[\frac{y(i)}{y(i - 1)} \right] + c_2 \ln \left[\frac{y(i - 1)}{y(i - 2)} \right] + c_3 \ln \left[\frac{y(i - 2)}{y(i - 3)} \right] + \varepsilon_i, \quad (10)$$

where the additional lags are meant to capture the possibility that annuity factors are not adjusting instantaneously (i.e. on the same week) to changes in interest rates.

TABLE #2 HERE

Table #2 provides summary statistics for the regression specified by equation (10). Note that although the c_1 (zero lag) coefficient was statistically significant, the highest value and sensitivity was found for the c_2 (one week lag) coefficient. Even the c_2 (two week lag) coefficient was significant. The adjusted R-squared was above 67% for both males and females which confirms that interests are the main driver of annuity factors and payouts on an ongoing basis.

To understand why the one-week lag provided the greatest impact, we remind the reader that our data is collected on Wednesday afternoon while the bond yields are closing values at day end. It is quite conceivable that insurance companies are using ‘stale’ yield-curves, by a few days, to price these annuities. But once a week or two has gone by they have all updated their prices to reflect changes in the curve. Alternatively, we do find systematic deviations in the practices of various insurance companies, with some adjusting quickly (within a day) to changes in interest rates, while others lag by a few days or keep the same price for even longer periods of time. Averaging the best five quotes between ‘slowly’ and ‘rapidly’ adjusting companies might impact our results on what drives δ .

5.3 The Risk in Self-Annuitizing

It is important to stress that there is absolutely no guarantee that if the individual decided to self-annuitize at age $x = 65$ and actually earn an internal rate of return greater than our index value δ_{65}^{10} , they will be able to purchase the same exact life annuity stream at age $x = 75$. Indeed, interest rates might decline and/or the company might decide to change (worsen) their annuity pricing basis within that ten year ‘waiting’ period. It is not even clear if δ_{65}^{10} is unbiased estimate (expectation) of the random variable on an ‘expected return’ basis. A full investigation of random portfolio returns in life insurance portfolios – as originally suggested by Boyle (1976) – is beyond the scope of this article. In brief, the stochastic analogue of the differential equation (2) would be

$$d\widetilde{W}_t = \left(\delta\widetilde{W}_t - \frac{w_0}{a_x^\tau} \right) dt + \sigma\widetilde{W}_t dB_t, \quad (11)$$

and the retiree would face the random prospect that:

$$\Pr \left[\widetilde{W}_\tau < \frac{w}{a_x} a_{x+\tau} \right], \quad (12)$$

which represents the case in which the investment portfolio does not contain sufficient funds to purchase the exact same annuity. Once again, we refer the interested reader to the recent papers by Albrecht and Maurer (2002) as well as Young (2003) or Gerrard, Haberman and Vigna (2003) for a detailed exploration of these and related issues.

5.4 Quadratic Approximation to Delta.

Looking back at equation (5) we can approximate the exponential term $e^{\delta\tau}$ over small values of δ with the quadratic form $(1 + \delta\tau + 0.5(\delta\tau)^2)$. Using this approximation, and then collecting terms, the implied life credits is the value of δ that solves:

$$- \left(\frac{1}{2} a_x^\tau \tau^2 \right) \delta^2 + \left(\frac{1}{2} \tau^2 - a_x^\tau \tau \right) \delta + (a_{x+\tau} + \tau - a_x^\tau) \approx 0. \quad (13)$$

The solution to this quadratic equation in δ is:

$$\delta^* \approx \frac{(\tau - 2a_x^\tau) + \sqrt{\tau^2 + 4a_x^\tau(\tau + 2a_{x+\tau} - a_x^\tau)}}{2\tau a_x^\tau}. \quad (14)$$

For example, in our earlier case (Male 65) for which $a_{65}^{10} = 12.2871$ and $a_{75} = 8.5391$ the exact value of the ILC is $\delta^* = 5.900\%$ using the NeRa method. Yet, according to the approximation in equation (14) we obtain:

$$\delta^* \approx \frac{(10 - 24.5742) + \sqrt{100 + 49.1484(10 + 17.0782 - 12.2871)}}{2(10)(12.2871)} = 0.05771,$$

which is an ILC value of 5.771%, a mere 12 basis points lower than the true value. Or, in the joint-life case mentioned above, the precise value of the ILC was $\delta^* = 5.21\%$ while the approximate value is $\delta^* = 5.14\%$. It seems our quadratic approximation consistently underestimates the true value of δ^* by between 10 and 20 basis points.

FIGURE #4 HERE

To understand the nature of this approximation, Figure #4 illustrates the *true* versus the *approximate* value of the left-hand side of equation (5), as well as the point (root) at which the two functions hit zero. The numerical parameters are $a_{65}^{10} = 12.2871$, $a_{75} = 8.5391$ and $\tau = 10$. Note that for small values of δ , the two graphs are visually indistinguishable. Heuristically, the quadratic approximation is an inverted parabola that crosses the y-axis at a positive value of $(a_{x+\tau} + \tau - a_x^\tau)$ and whose second root is the ILC we are interested in calculating. In the above case, both graphs cross zero within 10 basis points of each other. The two functions do start to diverge quite rapidly once δ reaches larger values, as one would expect from a quadratic (a.k.a. Taylor) approximation. To the left of the root δ^* , the δ value is not high enough to beat the life annuity, while to the right of the root δ^* , one earns more than enough to acquire the same annuity at age 75.

6 CONCLUSION

A number of recent papers have examined the stochastic properties of a *consume-term and invest-the-difference* strategy that self-annuitizes instead of purchasing an irreversible life annuity at retirement. In this brief paper we have used the idea as the basis for constructing a life annuity index to track and explain life annuity payouts over time. And, while the concept underlying the rate of return from a life annuity has been investigated in a number of actuarial and insurance papers such as Broverman (1986), our index does not require any knowledge of the company's mortality tables, rates, loads or pricing basis.

In fact, it might actually represent a more robust method of calibrating and monitoring the change in Money's Worth of a generic life annuity, in contrast to the highly-cited work by Mitchell, Poterba, Warshawsky and Brown (1999). Namely, by tracking the ILC values over time and across countries, one can obtain a better measure of the investment return from annuities without having to make distributional assumptions regarding mortality or relying on the accuracy of a given day's yield curve.

We calibrated our index to a comprehensive database containing the last three years of annuity quotes from insurance companies in Canada, and confirmed the accuracy of an easy-to-use approximation to the ILC which is presented in equation (14). Aside from describing the mechanics of such an index, our main actionable conclusion is that a 65-year-old retiree

would have had to earn at least 50 to 100 basis points over the yield on a risk-free 10-year Government bond to 'beat' the rate of return from a life annuity during the 2000 to 2003 period. Ongoing research will monitor the ILC values over longer periods of time and in different countries.

References

- [1] Albrecht, P. and R. Maurer (2002), Self-annuitization, consumption shortfall in retirement and asset allocation: The annuity benchmark, *Journal of Pension Economics and Finance*, Vol. 1(3): 269-288.
- [2] Ameriks, J. R. Veres and M. J. Warshawsky (2001), Making retirement income last a lifetime”, *Journal of Financial Planning*, December: 1-14.
- [3] Blake, D., A.J.G. Cairns and K. Dowd (2003), Pensionmetrics 2: stochastic pension plan design during the distribution phase, *Insurance: Mathematics and Economics*, Vol. 33: 29-47.
- [4] Davidoff, T., J. Brown and P. Diamond (2003), Annuities and individual welfare, *NBER Working Paper 9714*, May 2003.
- [5] Boyle, P.P. (1976), Rates of return as random variables, *Journal of Risk and Insurance*, Vol. 43(4): 693-713
- [6] Broverman, S. (1986), The rate of return on life insurance and annuities, *Journal of Risk and Insurance*, Vol. 53(3): 419-434
- [7] Carriere, J.F. (1994), An investigation of the Gompertz law of mortality, *Actuarial Research Clearing House*, Vol. 2: 1-34
- [8] Carriere, J.F. (1999), No-arbitrage pricing for life insurance and annuities. *Economics Letters*, Vol 64: 339-342.
- [9] Frees. E.W., J. Carriere and E. Valdez (1996), Annuity valuation with dependent mortality, *The Journal of Risk and Insurance*, Vol. 63(2): 229-261
- [10] Dushi, I. and A. Webb (2003), Annuitization: Keeping your options open, *Working Paper International Longevity Center*.
- [11] Gerrard, R., S. Haberman and E. Vigna (2003), The income drawdown option *Proceedings of the Seventh International Congress on Insurance: Mathematics and Economics*, Lyon.

- [12] Kapur, S. and M. Orszag (1999), A portfolio approach to investment and annuitization during retirement, *Birkbeck College Mimeo*, May 1999.
- [13] Khorasanee, M.Z. (1996). Annuity choices for pensioners, *Journal of Actuarial Practice*, Vol. 4(2): 229-255.
- [14] Milevsky, M.A. and C. Robinson (2000), Self-annuitization and ruin in retirement, *North American Actuarial Journal*, Vol. (4): 113-129.
- [15] Mitchell, O. S., J. M. Poterba, M. J. Warshawsky, and J. R. Brown (1999), New evidence on the money's worth of individual annuities, *American Economic Review*, Vol. 89(5): 1299-1318.
- [16] Milevsky, M.A. (1998), Asset allocation towards the end of the life-cycle, to annuitize or not to annuitize? *Journal of Risk and Insurance*, Vol. 65(3): 401-426.
- [17] Reichenstein, W. (2003), Allocation during retirement: adding annuities to the mix, *Journal of the American Association of Individual Investors*, November: 3-9.
- [18] Yaari, M. E. (1965), Uncertain lifetime, life insurance and the theory of the consumer, *Review of Economic Studies*, Vol. 32: 137-150.
- [19] Yagi, T. and Y. Nishigaki (1993), The inefficiency of private constant annuities, *Journal of Risk and Insurance*, Vol. 60(3): 385-412.
- [20] Young, V. (2003), Optimal Investment Strategy to Minimize the Probability of Lifetime Ruin, *Working Paper*, University of Michigan.

TABLE #1

	Delta Male	Delta Female	Can. Gov. 10yr Bond Yield	Delta Spread off 10yr Bond Male	Delta Spread off 10yr Bond Female
AVERAGE	6.374%	5.956%	5.237%	1.137%	0.719%
MIN	5.455%	5.039%	4.180%	0.782%	0.358%
10%	5.828%	5.374%	4.783%	0.924%	0.490%
25%	6.180%	5.739%	4.958%	1.037%	0.610%
50%	6.476%	6.062%	5.295%	1.125%	0.697%
75%	6.637%	6.216%	5.490%	1.246%	0.827%
90%	6.723%	6.310%	5.717%	1.364%	0.953%
MAX	6.831%	6.408%	5.960%	1.639%	1.195%

This table displays the summary statistics for the Implied Life Credits (ILC) index over the period Oct/2000 to Nov/2003 for both males and females at age 65 against age 75. We compare this number against the yield on a 10-year Government of Canada bond.

Table #2

Regressing Changes in ILC values on Changes in Bond Yields:

		MALE		FEMALE	
Multiple R		0.678		0.669	
R-squared		0.459		0.448	
Adjusted R-squared		0.449		0.437	
			<i>T-stat.</i>		<i>T-stat.</i>
c0	Intercept	-0.00055	-0.972	-0.00061	-1.051
c1	No lag	0.07212	2.808	0.09826	3.813
c2	One week lag	0.27575	10.68	0.25454	9.827
c3	Two week lag	0.05744	2.22	0.07752	2.987

FIGURE #1

Life Annuity Payouts per \$100,000 premium *Male 65 with 10yr Payment Certain*

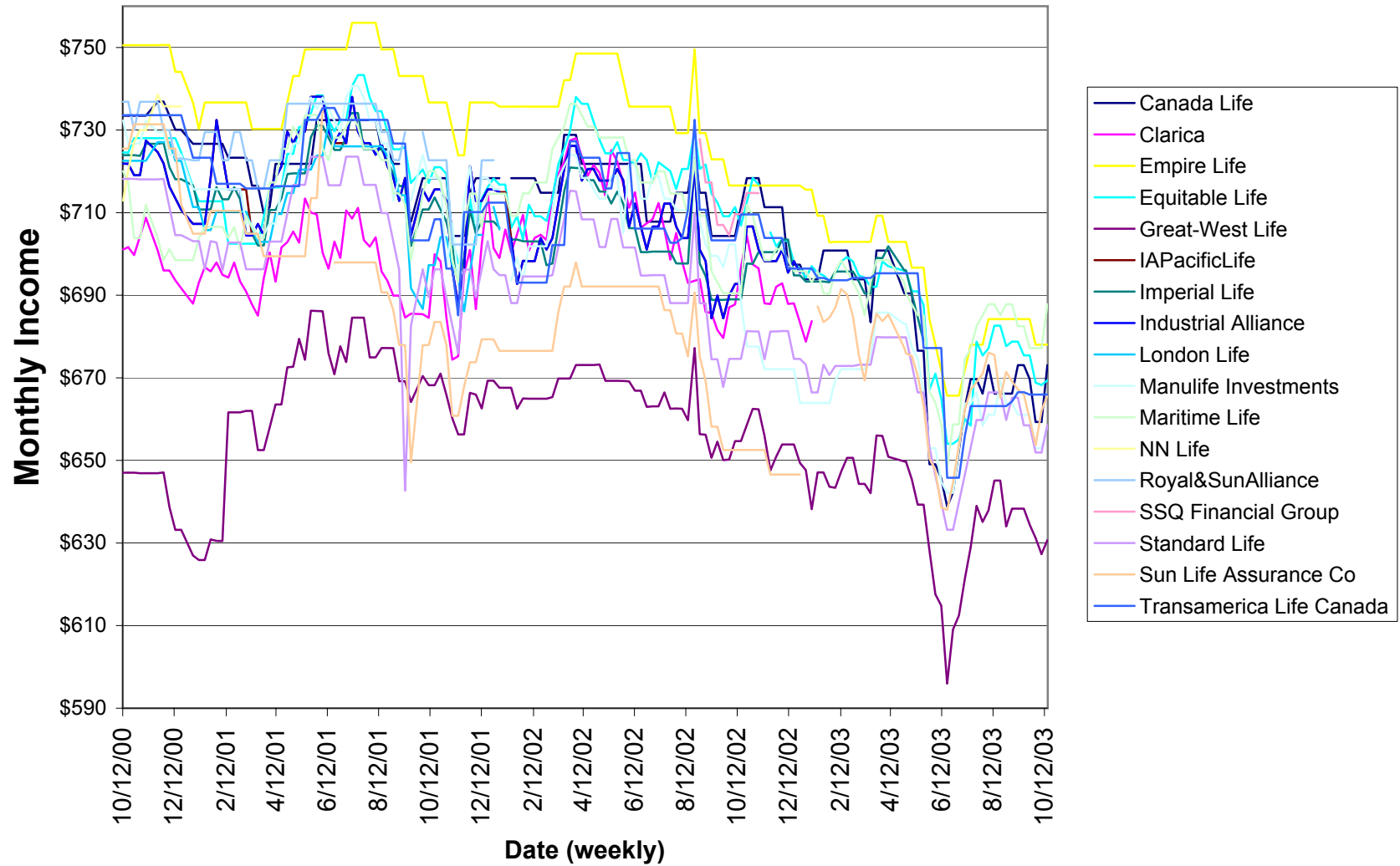


FIGURE #2

Implied Life Credits vs. 10yr Gov. Bond Yield

- Male Delta
- Female Delta
- 10yr Bond Yield

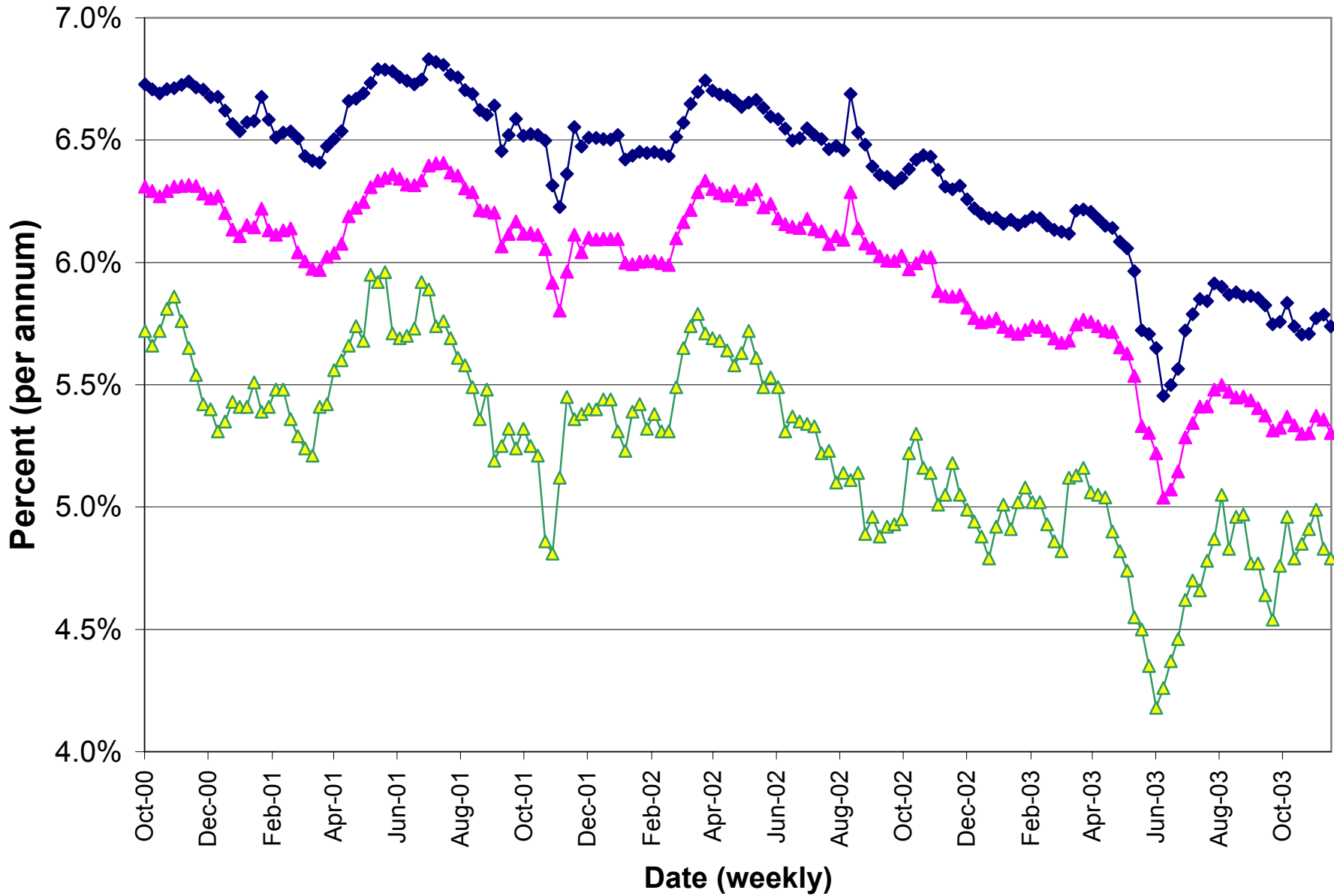


FIGURE #3

I.L.C. spread over 10yr Gov. Bond Yield

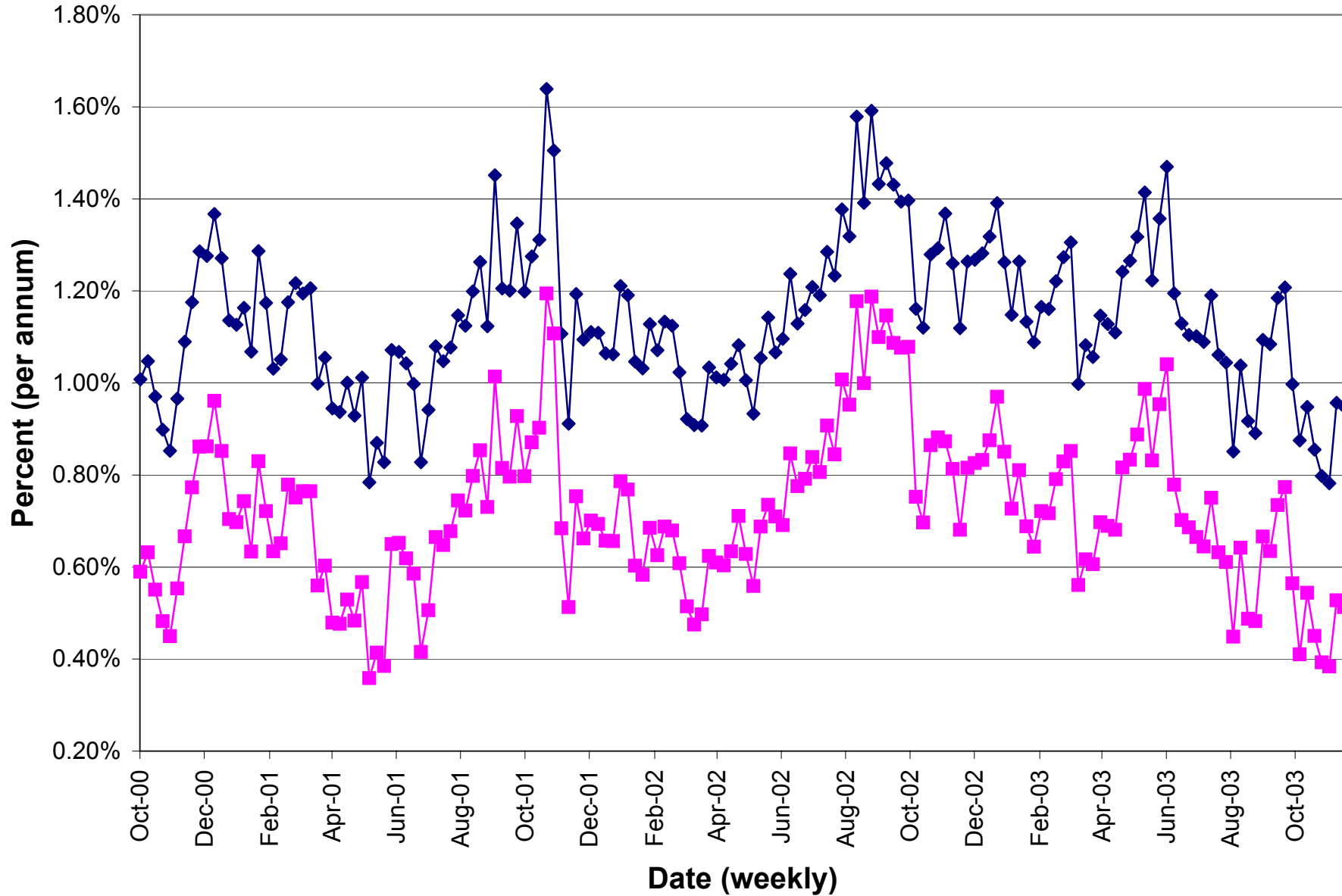
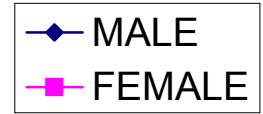


Figure #4

Exact vs. Approximate Solution to I.L.C.

