Illiquid Asset Allocation and Policy Weights: How Far Can They Deviate?

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Abstract:

Most investors, both institutional and individual, have strict policy benchmarks which dictate the ranges over which asset allocations can deviate in practice. In order to stay within these policy weights, managers must rebalance on a regular basis by selling-off the relative winners, and giving these funds to the relative losers. However, commitments to alternative asset classes such as venture capital and private equity – collectively labeled illiquid investments -- can severely impede the necessary rebalancing process since they can not be easily sold. Indeed, some argue that this is precisely why these asset classes provide superior returns. Yet an investor might find a 5% allocation mushrooming into a 20% commitment during periods of market volatility. And, while this might be good news from a valuation perspective, the fund will find itself consistently violating its documented policy benchmarks.

Motivated by this problem, the paper develops and illustrates a relatively simple model to compute the probability that an initial asset allocation will breach a pre-specified policy-weight over a given time horizon. This model is consistent with assumptions made in most Asset Liability Modeling (ALM) studies. Furthermore, our closed-form analytic expression 'buys' the user a variety of robust insights and risk metrics that are quite easy to compute and use.

After calibrating this model to a broadly defined "alternative investment" asset class data, we conclude that a conservative 5% commitment to an illiquid asset class has a 1/3 chance of doubling (i.e. to 10% of the fund) within 5 years, and tripling (i.e. to 15% of the fund) within 15 years. Paradoxically, the lower the effective correlation between the performance of a given asset class and the remainder of the portfolio – which is normally something to be coveted in strategic asset allocation – the greater the chances of breaching any given policy weight.

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Introduction and Motivation.

A recent article in the *New York Times* (21 March 2004) described the plight of the Louisiana teachers' pension fund which had originally established a policy weight of 18% of assets for alternative investments, but then found itself with a mushrooming 42% allocation over time without the ability to rebalance or sell-off ownership in many of these private partnerships and return the fund's allocation to the policy limit. And, while an increasing value for any investment might be viewed as a positive development – if, as expected, the alternative investment component has outperformed the traditional component -- drifting too far from the policy benchmark can prove disastrous when markets fail to "deliver the expectations" as Arnott and Plaxco emphasize in a recent article in the *Journal of Portfolio Management* (2002)**.**

Indeed, alternative investments have been growing in popularity over the last few years and while the average State pension fund policy allocation to these alternative investments is approximately 4.7% (according to Wilshire Associates) a number of states such as Washington, Michigan, Minnesota and Pennsylvania have more than 10% committed or allocated to these categories, most of which is in the form of highly illiquid partnerships.

Motivated by this issue, our paper develops a model to quantify the probability that an initial asset allocation will breach a pre-specified policy-weight over a given time horizon. This knowledge should enable managers to *a priori* compute the odds that a (comfortable) initial allocation grows to an undesired position over time. If the probability (or risk) of this event is pre-judged to be unacceptable, then perhaps a lower (and perhaps even suboptimal) policy allocation should be set in advance, in anticipation of a higher level being breached in the future.

This model is obviously predicated on a number of simplifying assumptions regarding the dynamic evolution of the illiquid asset class, foremost of which is that the investment can not be liquidated during the horizon in question. On the other hand, our closed-form analytic format 'buys' the user a variety of robust insights and risk metrics that are quite easy to compute.

A number of recently published articles in the institutional portfolio management literature, such as Masters (2003), Singer et al (2003), Arnott et al. (2002), Buetow et al.

(2002), and Takahashi et al. (2002) have discussed the problems created by the lack of rebalancing. Yet, none have specifically derived an easy-to-use expression for the probability various policy levels will be breached, nor have they provided much discussion of the "high level" analytics of this problem, which is the contribution of this paper.

High-level Description of the Model.

Of course, one way to investigate the question of "How far can the allocation weights deviate over time?" is to conduct extensive computer (Monte Carlo) simulations of the future projected cash-flows of the illiquid component and to compare them – scenario by scenario – to the market value of the liquid or traditional component. Running a large number of these simulations would provide us with a statistical distribution of the probability the illiquid component grows to more than x% of the fund. In contrast to a 'brute force' approach our analysis takes a subtler path by knowingly and deliberately ignoring the micro issues of capital calls, takedowns and commitment schedules and focusing instead on the high-level dynamics of the asset class. Like assumptions made in Asset Liability Modeling (ALM) studies, we assume that the unknown future IRRs, exhibit a jointly lognormal distribution over the life of the commitment. As such, our analysis parsimoniously meshes with standard capital market assumptions used to generate Markowitz efficient frontiers and probability distributions for future pension contribution rates and funded ratios. The closest analogy to the two different approaches to dealing with this problem is the current methodology for valuing call and put options. One can simulate thousands of scenarios for the evaluation of the underlying security (under the relevant pricing measure), and then discount the expected cash-flow to obtain a 'theoretical value'. Or, if the user is willing to rely on the log-normal assumption of asset prices, they can employ the celebrated Black-Scholes formula that depends on a limited set of input parameters. Another benefit of this 'high level' approach is that our model – which is described fully in the next section -- can be used easily and repeatedly with any range of input variables to test the sensitivity of assumptions. This would obviously be very time consuming within the context of a simulation study.

Of course, all models requite input parameters and in this paper, we have settled upon the following economic parameter estimates.

1. The model requires as input the difference between the expected investment return on the illiquid asset class and the expected return from the conventional (fully liquid) asset class. Note that the model does not require a precise estimate of the actual returns themselves, only the difference between them. This is because we are only interested in the relative out-performance of one asset class over the other. The number we picked for the subsequent examples was 450 basis points per annum, which is in line with various institutional policy benchmarks for alternative investments as per a recent article in *Pension and Investments (February 2004)*.
2. The volatility (or standard deviation) of the investment return on the illiquid asset component was assumed to be $\sigma_A = 35\%$. This number is consistent with a recent empirical study Bergman and Howard (2003) in the *Journal of Wealth Management* as well as a number of ALM studies that have optimized alternative asset allocations. Although, we should note that recent academic studies (University of Chicago working papers) have raised concerns regarding the presence of survivorship bias in the published data series for venture capital and private equity and claim that true (unbiased) volatility might be much higher than assumed in any of the above sources.
3. The volatility (or standard deviation) of the investment return for the conventional (liquid) component – which comprises of a weighted average of domestic and international equity plus bonds fixed income and real estate – was assumed to be σ_M = 18%. This number is based on the traditional *Ibbotson Associates* estimates and consistent with other sources such as the above-mentioned Bergman and Howard (2003) study.
4. The correlation between the Internal Rate of Return (IRR) earned on the illiquid portion and the return from the conventional portion was estimated

to fall within a range of 40% to 50%, and so a midpoint estimate of $\rho = 45\%$ was used.

Note that the correlation estimate is the most problematic input to the model given the wide range of defensible numbers. In fact, in a recent article in the *Journal of Alternative Investments* by Milner and Vos (2003), the authors used quarterly performance data across a 10-year period (1991-2001) for eight listed equity benchmarks to compute correlations with eight different private equity fund categories. They found a wide range of 'empirical or realized correlations'. For example Venture Seed/Start-up, Mezzanine and Private Equity Special Situation funds were uncorrelated with broad-based equity $\{\rho = 0\}$, while Venture Early Stage, Venture Late Stage, Venture Balanced and Buyout funds were positively correlated with the market $\{\rho > 0\}$.

In any event, the nice and convenient aspect of our model is that the user can select any correlation and easily obtain the relevant results.

Technical Model

We denote the so-called market value of the illiquid, a.k.a. *alternative investment* (illiquid asset) portion of the fund by the symbol A_t and the market value of the remaining conventional asset portion by the symbol M_t. Initially, the target policy allocation to the illiquid asset is set to and denoted by $\alpha^* = A_0/(A_0 + M_0)$; for example 5%. Define the logarithmic ratio of the illiquid asset allocation to the conventional asset allocation – which is a key variable in our analysis – by the symbol:

$$R_t = \ln\left[\frac{A_t}{M_t}\right], \quad R_0 = \ln\left[\frac{\alpha^*}{1-\alpha^*}\right] \qquad \text{(eq.1)}$$

For example, with a target illiquid asset allocation of $\alpha^* = 5\%$, the initial value of this key ratio is $R_0 = -2.944$ units. This variable will obviously change over time, but does not necessarily have any specific meaning in its own right. For example, if R_t were ever to hit a value of zero, it would imply that the illiquid asset allocation has breached 50% of the value of the fund; since $\ln[0.5/0.5] = 0$, etc. From a mathematical point of view this transformation is a one-to-one mapping that proves convenient in our analysis.

We now make the traditional, yet critical, assumption that the ongoing value of the illiquid asset portion of the fund $\{A_t\}$ and the market value of the remainder of the fund $\{M_t\}$, are jointly log-normally distributed. As mentioned earlier, this assumption is the theoretical bedrock of most Asset Liability Management (ALM) models and implies that continuously compounded investment returns are joint normally distributed. And, while the normality assumption has been criticized for years by scholars – and are clearly inconsistent with large market movements over short periods of time – we feel it is a reasonable approximation to actual market behavior over the periods of time relevant for this analysis. Moreover, we can easily generalize our approach to time-dependent risk-premiums and volatility, if needed. In either event, the dynamics of $\{A_t\}$ and $\{B_t\}$ obey a geometric Brownian motion (GBM) which is the continuous-time analog to the log-normal distribution, and specified by the stochastic differential equation (SDE):

$$\begin{aligned} dA_t &= \mu_A A_t dt + \sigma_A A_t dB_t^{\,A} \\ dM_t &= \mu_M M_t dt + \sigma_M M_t dB_t^{\,M}, \end{aligned} \qquad \text{(eq.2)}$$

where the parameters $\{\mu_A, \mu_M\}$ denote the expected return from the two asset classes (for example 11.3% and 6.8% respectively, per annum) and the parameters $\{\sigma_A, \sigma_M\}$ denote the standard deviation of volatility of the asset class returns (for example 35% and 18% respectively). The two processes are 'driven' by correlated Brownian motions $\{B_t^{\,A}, B_t^{\,M}\}$, where the correlation coefficient is denoted by $\{\rho\}$. Theoretically, this input parameter can range anywhere from the extreme of *minus* 100% to *positive* 100%; both of which are quite rare. In practice, we estimated the correlation between returns on the illiquid asset and returns on the conventional asset to be in the vicinity of 40% to 50%, and so we used a midpoint of 45% in the numerical results displayed in Table #1 and Table #2 in the next section.

Now, using a variant of what is known as *Ito's formula*, we can write down an expression for the dynamical evolution of our key ratio $\{R_t\}$, which can be expressed as:

$$\begin{aligned} dR_t &= (\mu_A - \mu_M - (\sigma^2{}_A - \sigma^2{}_M)/2)dt \\ &+ (\sqrt{\sigma^2{}_A + \sigma^2{}_M - 2\sigma_A\sigma_M\rho})dB_t, \quad R_0 = \ln[\alpha^* /(1-\alpha^*)]. \end{aligned} \qquad \text{(eq.3)}$$

Note that our key ratio is only a function of the spread between the expected returns $\{\mu_A - \mu_M\}$ and not the rates individually. This will prove quite helpful in our model's calibration since it avoids requiring an estimate of the market benchmark, and only requires the illiquid asset premium. Note, also, that the volatility of the key ratio $\{\sqrt{\sigma^2{}_A + \sigma^2{}_M - 2\sigma_A\sigma_M\rho}\}$, is a decreasing function of the correlation between the returns on the two asset classes. This means that the greater the co-movement between the (estimated) value of the alternative investment portion versus the traditional portion, the lower the volatility of this key ratio. This fact should not come as a surprise since the key ratio will deviate from its desired value (for example -2.944) only if the two portions of the fund wander-off in their own separate directions. If the valuations stay relatively close over time (i.e. the correlation is close to 100%) the key ratio will rarely deviate from its initial value. In fact, if the correlation between returns is 100%, the variance of the key ratio will be equal to the difference between the individual variances $(\sigma_A - \sigma_M)^2$. And, if they happen to be equal, the key ratio will never deviate from its expected value. An interesting point to note is that the portfolio volatility of the entire fund $A_t + M_t$, will be related to the expression $\sqrt{\sigma^2{}_A + \sigma^2{}_M + 2\sigma_A\sigma_M\rho}$, with a positive sign in-front of the correlation term. Thus, the overall portfolio volatility will be reduced when the correlation between the illiquid asset and conventional asset components is lower. But, the volatility of the key ratio will be higher. To simplify the clutter in our main formula, we abbreviate and denote the relevant drift and diffusion quantities by:

$$\mu_R := \mu_A - \mu_M - (\sigma^2{}_A - \sigma^2{}_M)/2), \qquad \text{(eq.4)}$$
$$\sigma_R := \sqrt{\sigma^2{}_A + \sigma^2{}_M - 2\sigma_A\sigma_M\rho},$$

so that equation (eq.3) can be simplified and written in shorthand:

$$dR_t = \mu_R dt + \sigma_R dB_t, \qquad \text{(eq.5)}$$

which is now easily identifiable as a (non-standard) Brownian motion with drift.

Recall that our objective is to compute the probability the illiquid asset allocation fraction $\{A_t/(A_t + M_t)\}$ or key ratio $\{\ln[A_t/M_t]\}$ will substantially deviate from a given initial value. For example, we would like to compute the probability that a 5% allocation becomes a (non-tolerable) 10% or 15% allocation over some time horizon. This is mathematically equivalent to the probability that the key ratio we introduced above will start off at an initial policy value of $\ln[0.05/0.95] = -2.94$ and wander off to hit a value of $\ln[0.10/0.90] = -2.19$ or $\ln[0.15/0.85] = -1.73$, for example. We are interested in:

$$\begin{aligned} &\Pr[\max_{0\le t\le T} \frac{A_t}{M_t} \ge \frac{\alpha}{1-\alpha}] \\ &= \Pr[\max_{0\le t\le T} R_t \ge \ln[\frac{\alpha}{1-\alpha}] \mid R_0 = \ln[\frac{\alpha^*}{1-\alpha^*}]], \end{aligned} \qquad \text{(eq.6)}$$

where $\{\alpha\}$ is any asset allocation level which is greater than the policy target $\{\alpha^*\}$, and $\{T\}$ is the horizon over which this is measured. In words, (eq.6) is the probability that the maximum value of the key ratio ever hits some pre-specified upper bound over a given time horizon.

Since the uncertainty driving our key ratio is a linear transformation of a standard Brownian motion, the probability we require in Equation (eq.6) can actually be derived using techniques from the theory of stochastic processes. See Karlin and Taylor (1975, ch7) for a derivation based on the first passage time properties of a Brownian motion. Indeed, we can scale $\{R_t\}$ to the origin by subtracting the initial value $R_0 = \ln[\alpha^*/(1-\alpha^*)]$ and then measuring the distance to be traveled to reach a given level. The probability is equal to:

$$\Pr[\max_{0\le t\le T} R_t \ge \ln[\frac{\alpha}{1-\alpha}]] = 1 - \left(\frac{L-\mu_R T}{\sigma_R\sqrt{T}}\right) + e^{2L\mu_R/\sigma^2}\left(\frac{-L-\mu_R T}{\sigma_R\sqrt{T}}\right), \qquad \text{(eq.7)}$$

where (x) denotes the cumulative standard normal variate evaluated at *x*, and $L := \ln[\alpha/(1-\alpha)] - R_0$. Equation (eq.7) is a function of the basic inputs variables named above. It depends on the drift and diffusion coefficients $\{\mu_R, \sigma_R\}$ of the key ratio process, on the time horizon $\{T\}$ and the 'distance to be traveled' $\{L\}$. Equation (eq.7) might also seem familiar to option pricing specialists since it is related to an exotic derivative known as a 'look-back option' that pays off based on the maximum value of the stock or index over the life of the option.

Here is a basic numerical example which is a prelude to the extensive numerical results in the next section. Assume the illiquid asset class is expected (in the arithmetic mean sense) to earn benchmark returns plus 450 basis points, and the remainder of the fund (non illiquid component) is expected to earn benchmark returns. In this case $\{\mu_A - \mu_M = 0.045\}$. Note once again that our model does not require an estimate of the actual benchmark return. Assume the volatility of the illiquid asset class is $\{\sigma_A = 0.35\}$ -- which is in line with estimates from venture economics data as well as the recently published study in the *Journal of Wealth Management* -- and the volatility of the conventional asset class is $\{\sigma_M = 0.18\}$ per annum. Finally, we use a correlation coefficient of $\{\rho = 0.45\}$, which is based on the same published sources. Thus, according to equation (eq.4) we have that $\{\mu_R = 0\}$ and $\{\sigma_R = 0.3134\}$. We are interested in the probability that a 5% allocation to the illiquid asset will triple to 15% within $\{T = 3\}$. A tripling of the allocation implies that our key ratio goes from an initial $R_0 = \ln[0.05/0.95]$ to $\ln[0.15/0.85]$ at some point during the next three years. The probability is obtained from equation (eq.7) and is 4%.

This analytic representation, which is a function of small and manageable number of parameters, provides us with a wide number of quick applications. For example, we can invert equation (eq.7) – easily done using solver in Excel -- and locate the allocation $\{\alpha\}$ for which the probability is under a given risk tolerance level $\{\varepsilon\}$, for example. We did exactly this to obtain Figure #1. Alternatively, one can solve for a required drift rate $\{\mu_R\}$ – and the implicit return on the alternative asset class – that satisfies the risk tolerance level. Finally, the exact same technology can be used to obtain the probability a lower level will <u>not</u> be breached:

$$\Pr[\min_{0\le t\le T} R_t \ge \ln[\frac{\alpha}{1-\alpha}]] = \left(\frac{-L+\mu_R T}{\sigma\sqrt{T}}\right) - e^{2L\mu_R/\sigma^2} \left(\frac{L+\mu_R T}{\sigma\sqrt{T}}\right), \qquad \text{(eq.8)}$$

where L is defined in the same manner of $L := \ln[\alpha/(1-\alpha)] - R_0$, but $\{\alpha\}$ is now an asset allocation percentage that is lower than the initial value $\{\alpha^*\}$.

Numerical Results

With the above model, input parameters and estimates in hand, Table #1 provides extensive output of the probability that an initial allocation of 5% to alternative investments will breach various levels over a given time horizon. For example, according to equation (eq.7), during a 3 year period there is a 38% chance the market value of the allocation will reach (the upper extreme of the allowable range of) 8%. Over 5 years this number grows to 49%, and over 10 years the number exceeds 63%. Notice the rapid rate at which these probabilities increase with time.

TABLE #1	Current Policy Allocation to Alternative Investment (A.I.) Asset Class is:					5%
	Probability Actual Allocation Hits the Following Values During a Given Time Horizon					
Time (years):	6.0%	8.0%	10.0%	15.0%	20.0%	40.0%
3	72%	38%	19%	4%	1%	0%
5	78%	49%	32%	11%	5%	0%
8	83%	59%	43%	21%	11%	2%
10	85%	63%	48%	26%	16%	3%
15	87%	69%	56%	36%	25%	8%
20	89%	73%	62%	43%	32%	14%

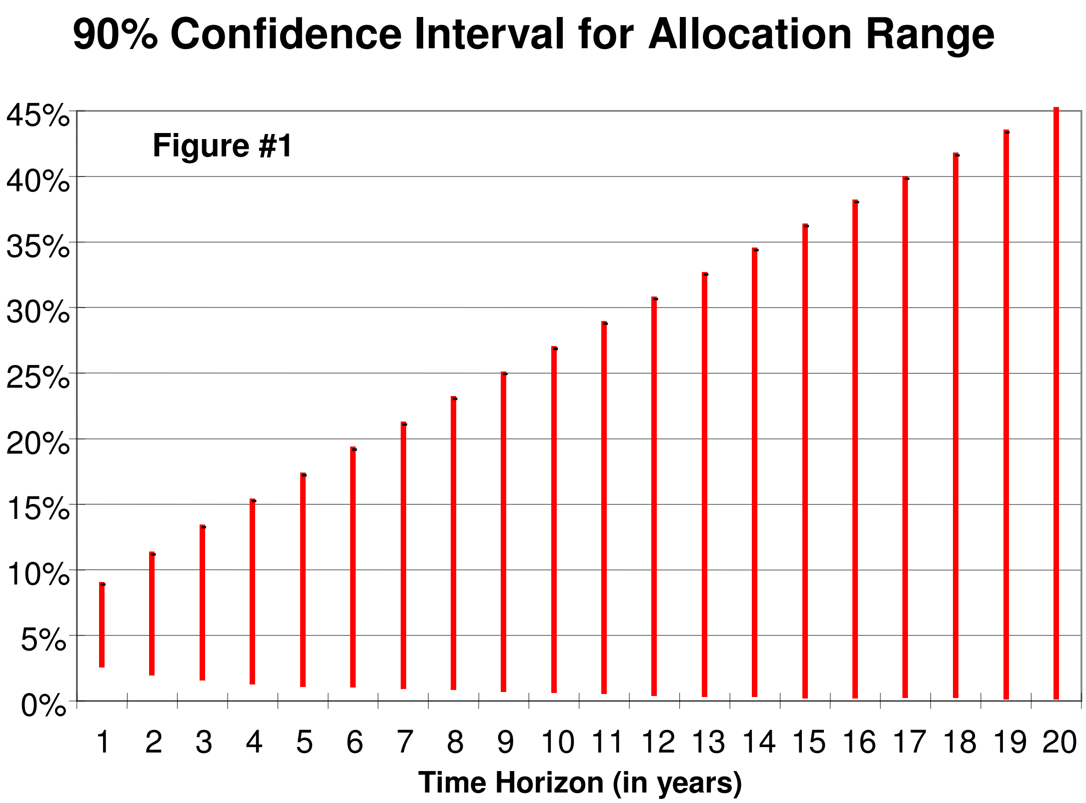
Table #2 provides similar estimates, but for a reduced 3% initial policy allocation to the illiquid asset. In this case, the probability that the actual allocation breaches the upper range of 8% of the fund's value is 6% within 3 years, 15% within 5 years and 31% within 10 years. Note that all the numbers displayed within these tables are computed directly from (eq.7). The input parameters are the expected return, volatility and correlation of the illiquid asset and conventional asset classes, the time horizon and the 'breaching' level of interest. The pricing model can be used for any horizon and/or policy allocation. Note that all numbers in both tables are rounded to the nearest percentage, which is why some entries appear as zeros, which should be interpreted as being less than 1%.

TABLE #2	Current Policy Allocation to Alternative Investment (A.I.) Asset Class is:					3%
	Probability Actual Allocation Hits the Following Values During a Given Time Horizon					
Time (years):	6.0%	8.0%	10.0%	15.0%	20.0%	40.0%
3	18%	6%	2%	0%	0%	0%
5	30%	15%	8%	2%	1%	0%
8	41%	25%	16%	6%	3%	0%
10	46%	31%	21%	10%	5%	1%
15	55%	40%	31%	18%	11%	3%
20	60%	47%	38%	24%	17%	6%

The analytic representation of the hitting (or breaching) probability allows us to 'invert the equation' and compute the <u>maximum</u> initial allocation that is consistent with a pre-determined risk tolerance. For example, if the pension fund is at most willing to tolerate (i.e. observe or experience) a 20% allocation to the illiquid asset over the next 10 years with a 5% probability, then the initial allocation can not exceed 3% since under this initial allocation – according to Table #2 -- there is a 5% chance that the allocation will grow to 20% of the fund during the next 10 years.

Finally, Figure #1 displays a (two-sided) 90% confidence interval for the evolution of the illiquid asset portion of the fund assuming the initial allocation is precisely 5%. These confidence intervals were obtained by solving for the breaching level in equation (eq.7) and (eq.8) which result in probabilities of 5% and 95% respectively.

Note the very wide range for the 90% confidence interval in the graph. Even after one year, the interval for possible observed values during the period ranges from an upper bound of 8.8% to a lower bound of 2.8%. And, during a five year period, the 90% confidence interval starts at 1.3% and goes up as far as 17% of the value of the fund. Just to be clear, the 90% interval means that there is 95% chance the illiquid asset allocation will be less than 17%, and a 5% chance it will be higher than 1.3% during the next five years. The mean value is indicated by the light dot within the bar. Although it is hard to see this from the bar-chart, the distribution of the range of possible illiquid asset allocation percentages is non symmetric. The expected value of the illiquid asset allocation at any given horizon grows from the initial 5% values by approximately 0.3% each year and reaches a mere 7.6% in year 10, and 9.4% in year 15. Yet, at the same time, the 90% interval covers a much wider range of 0.8% to 26.2% during the 10 years and 0.4% to 35% during the 15 years. This might seem counter-intuitive at first. After all, if the average value of the illiquid asset allocation is less than 8% after 10 years, why is the range so extreme? Of course, the answer to this lies in the difference between the expected trend of the illiquid asset allocation at a given point – which grows slowly over time – and the ever increasing range of uncertainty during this time horizon.



Recall that in the above chart and tables, the standard deviation (or volatility) of the illiquid asset allocation is 35% (and we feel this is a conservative estimate). Therefore, although the illiquid asset is expected to outperform the conventional asset class by 4.5% per annum, there is a fairly substantial chance it will beat the conventional asset class by a much higher margin. This leads to a wide range of outcomes which in turn increases the probability of hitting a very extreme allocation at some point during the horizon. In sum, the maximum value of a process grows at a much higher rate than the average value of a process, which is why the expectation for the illiquid asset allocation over the next 10 to 15 years remains within a comfortable bound, yet the probability of breaching extreme levels is so high.

Conclusion and Potential for Further Work

This paper has developed a simple analytical model for calculating the probability that an illiquid asset allocation that can not be easily traded, will breach a given (desired) policy level. We apply this model to an asset class where the ability to rebalance is difficult, if not impossible. As input, we require the same parameters and variables that are used in most Asset Liability Management (ALM) studies for large pension plans and thus can be placed in the same framework of analysis. Our model – which uses relatively simple ideas from the theory of stochastic processes – provides us with a number of practical insights that can be applied in a number of real-world situations.

- We find that a 5% initial commitment to an illiquid asset – which is the average policy allocation to illiquid assets by a number of large institutional investors -- which remains completely un-rebalanced relative to the conventional asset portion has a 32% probability of wandering to 10% of the fund within 5 years and a 36% probability of growing to 15% of the fund within 15 years.
- The results from this analysis might argue in favor of reducing the *initial* policy allocation to illiquid asset in anticipation the actual policy weight will wander to that level (and ever further) over time. Thus, a 2% - 4% policy weight to an illiquid asset class might actually be a 5% to 8% when the future is properly taken into account.
- Correlation – the holly grail of diversification and portfolio theory -- has a counter-intuitive impact on the 'weight wandering' problem. While seemingly un-correlated and counter cyclical returns are the *raison detre* of the illiquid asset allocation, a low correlation between the illiquid asset and conventional asset greatly increases the probability of breaching extreme and undesirable levels. In fact, paradoxically, an increase in any of the beneficial characteristics of illiquid asset deals – greater expected IRRs over longer horizons -- leads to higher probabilities of breaching any pre-specified level.
- Thus, when contemplating allocations to an illiquid "deal" at the margin, one must be ever cognizant that the greater the diversification properties of this particular allocation, the higher the probability the policy weight will be breached during the life of the commitment. In some sense, they are conflicting objectives.

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