

APPLIED Risk Management During Retirement

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ABSTRACT:

*In this chapter we describe the 3+1 components of a prudent risk management strategy during retirement. First, we re-emphasize the importance and need for a balanced **a**sset allocation containing both equity and fixed income components, even towards the end of the life-cycle. Second we illustrate the impact of implementing **p**ortfolio **p**rotection, which minimizes retirement-ruin risk and increases the sustainability of the portfolio. Third, we explain the need for **l**ongevity **i**nsurance in the form of payout annuities, for those who do not have a substantial defined benefit pension. We also provide a formula that links all these risk factors together to produce a self-contained probability of whether a given retirement plan is sustainable or will lead to ruin. Finally, we emphasize the importance of **e**ducating the public about the unique risks faced by individuals during retirement. In sum, we present an APPLIED risk management strategy for retirees and their financial advisors.*

INTRODUCTION

Jorge Guinle -- the famous Brazilian playboy -- died on Friday, March 5th, 2004 in Rio de Janeiro. According the obituary which appeared in the *Financial Times*, Jorge was born to one of the wealthiest families in Brazil, and spent a large part of his life and disposable income dating famous Hollywood starlets such as Rita Hayworth, Lana Turner and Marilyn Monroe. This hobby was obviously quite expensive and he apparently squandered most of his family's fortune well before his death at the age of 88. His investment acumen wasn't that great either, since he is reported to have invested (and lost) millions in a variety of poorly executed business ventures. In fact, in an interview a few years before his death, Jorge said: "The secret of living well is to die without a cent in your pocket. But I miscalculated, and the money ran out too early."

Although Jorge was obviously a very colorful character and was not the typical client for a financial advisor, imagine for a moment that you had the opportunity to interview or speak to Jorge on his death bed and you asked him the following question: “Jorge, why did you run out of money? Is it because you spent too much, or is it because you did so poorly with your investments, or is it because you lived too long?” What do think Jorge would have answered? Obviously, all three factors contributed to his lousy situation, but which was the most critical factor?

This question – which might seem a tad crude to some -- is the essence of the retirement income dilemma. The main thesis of this chapter is that there are three basic financial risks outside of our immediate control that you or your clients face during the retirement spending phase of the life-cycle. The first is investment and portfolio risk, the second is consumption and inflation risk and the third is mortality and longevity risk. We will discuss each of these risks in detail and then link them all together using some ideas from probability and statistics. In other words, the objective of this chapter is to carefully examine the root causes of the unfortunate and undesirable "dying without a cent in your pocket" using a risk management framework. We begin with longevity risk.

Longevity Risk

Human beings have a random (and finite) lifespan and therefore any serious discussion concerning retirement planning must account for this uncertainty. Exhibit #1 and the corresponding Exhibit #2 illustrate the probabilities of survival based on mortality tables from the U.S.-based Society of Actuaries. For example, according to this particular mortality table (and there are hundreds of them) a 65 year-old female has a 34.8% chance of living to age 90, while a 65 year-old male has a 23.7% chance of living to age 90. The probabilities of survival decline in a roughly (faster than) exponential manner with age – as the graph in Exhibit #2 illustrates - reaching close to zero sometime

between ages 105 and 115 depending on the mortality table, projection method and gender. And, while the often quoted statistic for life expectancy is somewhere between 78 and 82 years in the U.S., this is only relevant at the time of birth. By the time pensioners reach their retirement years, they may be facing 25 to 30 more years with substantial probability, especially if the prior years of their lives have been healthy. From our retirement spending perspective, a 65 year-old might live 20 more years or 30 more years or only 10 more years. This is longevity risk.

Along the same lines, Exhibit #3 focuses on a couple, as opposed to an individual, and answers the following question: If a married couple is 65 years old, what is the probability that at least one of them survives from the current age 65 to age 90? In this case, there is a 50.3% chance that one, the other, or even both could survive for the next 25 years.

Clearly, the prospect of living a longer-than-average life is not unimaginable. In fact, throughout recent decades mortality has been steadily improving, confirming once again that the concept of longevity risk must be recognized, managed and incorporated into all retirement plans. Obviously, we are not the first – and will certainly not be the last – to emphasize the critical role of longevity risk in retirement income planning. Please refer to the chapter by Ibbotson, Chen and Henkel (2005) in this book, or the independent work by Ameriks, Veres and Warshwasky (2003), or Blake, Cairns and Dowd (2000) for a further discussion of longevity risk.

Inflation Risk

Another real risk source that presents a challenge for retirement planning is inflation or the rate of change of price levels as measured by the Consumer Price Index (CPI). Another way to think of this is as consumption risk, under a looser definition of risk that you will consume “too much” because prices have

increased by more than you expected. Exhibit #4 illustrates that even under a fairly benign inflation rate of 2% -- which some consider to be an ideal target of the Federal Reserve -- a \$1,000 income flow in today's dollars would only be worth \$610 in 25 years, in real terms. That is, by the end of the 25-year period the purchasing power of \$1,000 would drop by nearly 40%. In fact, we believe that un-anticipated inflation is a greater risk factor and concern for the retired elderly as opposed to the rest of the population. We are not alone in this belief. For the last 20 years the U.S. Bureau of Labor Statistics has compiled a unique inflation index called the CPI-E (for elderly), which is distinct in composition from the more common and popular CPI-W (or CPI-U) index. The retiree inflation index has outpaced the regular broad-based CPI during the last 20 years by almost 0.5% to 1% per year. As you can see in Exhibit #5, the difference in composition between the two indices arises because categories such as housing and medical care are assigned different weights, within the broad-based CPI-W and the unique CPI-E. Retirees typically consume more health-care services, but less food – both of which should come as no surprise. The lesson is clear for financial advisors and their newly retired clients. Think about YOUR inflation rate.

So, Will the Money Run Out?

Monte Carlo Simulations (MCS) are by-now a widely used technique for quantifying the risks of retirement spending and asset allocation strategies. See the papers by Ho, Milevsky and Robinson (1994) or Pye (2000) for some of the earlier work in this area. MCS enables financial advisors and their clients to answer the following types of questions: If a 65 year-old female allocates half of her \$100 nest-egg to stocks and half to bonds, and withdraws \$5 per year, what are the chances that she too meets Jorge's fate of running out of money before running out of life? Exhibit #6 shows one possible outcome based on a sequence of random returns. Under this particular sequence – and there are millions of possibilities, obviously – if she insists on consuming \$5 each year, she will run out of money in 20 years. And, there is nearly a 50% chance that she will still be

alive at that point, and so this is the probability of ruin for this scenario. If we run thousands of these scenarios – each with a unique return sequences – we can estimate the overall probability she will run out of money, while still alive.

Of course, we can vary a number of factors such as the age, gender, asset allocation and the fixed consumption rate, all of which will impact the probability of retirement ruin. As Exhibit #7 shows, a 65 year-old male, investing half of his \$100 nest-egg in equity and withdrawing a constant real \$5 per annum, has a 10.6% chance of running out of money within his lifetime. If she makes the same investment and consumption choices, a 65 year-old female has a 12.3% probability of retirement ruin – as is shown in Exhibit#8. The higher ruin probability makes intuitive sense, since on average females are expected to live longer than males. Notice that if the female decides to lower her fixed real annual consumption by \$1, her probability of retirement ruin drastically drops to 4.5%.

These numbers were generated using standard simulation techniques under a 7% expected return, 5% growth rate and 20% volatility of equity-based investments and a 2.5% expected return, 2% growth rate and 10% volatility for bond-based investments. Obviously these numbers are all provided in inflation-adjusted terms and are consistent with (but slightly lower than) historical returns, as documented by Ibbotson Associates (2004). The mortality tables used are similar to the numbers provided in Exhibit #1 and Exhibit #2.

Using the same inputs, if we age our retirees by 10 years so that the male and female begin withdrawing \$5 per year from their initial \$100 nest egg at age 75, the probabilities of retirement ruin drop, as demonstrated in Exhibits #9 and #10. For example, the 75 year-old male faces a ruin probability of 2.4%, and the 75 year-old female - a probability of 4.5%, under the same 50% equity allocation and \$5 consumption rate. Again, the reduced ruin probabilities can be attributed to reduced expected remaining years of life.

The main point of Exhibits #8, #9 and #10 is that regardless of age or desired consumption (spending) rates, a balanced portfolio provides the best odds. It is only at unrealistically high consumption rates that we see “corner” allocations providing the best odds, and that is likely a gambling strategy as opposed to a prudent investment strategy. Stated differently, for most reasonable spending patterns, an overly conservative asset allocation, as well as an overly aggressive asset allocation will only serve to increase the risk of running out of money too soon. Exhibit #11 shows this graphically, by illustrating the risk and return tradeoff for a male and a female, age 65, who consume \$6 per annum.

The Retirement Ruin Formula

Although the widely-used techniques of Monte Carlo Simulations can help shed light on sustainable withdrawal rates and optimal portfolios, it is not without problems. First, many thousands of simulations are needed to produce increasingly accurate and meaningful results, and results are difficult to replicate by others.

An alternative method is to use a simple analytic expression that captures the relationship between spending, aging and a random portfolio return, to estimate the probability that a given spending rate is sustainable. This formula brings together the longevity, spending and investment risk factors, under one unified umbrella. We call it the Exponential Reciprocal Gamma (ERG) formula, for reasons that will become evident in a moment. This is obviously not the time or place to delve into the derivation of this formula, so we refer the interested reader to Milevsky and Robinson (2005). In the next few paragraphs we provide some numerical examples of this formula, which can easily be implemented in Excel or any other spreadsheet. We find that it reproduces results that are typically within 5% to 10% of extensive Monte Carlo Simulations.

The formula – which is the probability of retirement ruin – can be computed as follows:

$$\Pr[Ruin] = \text{GammaDist}\left(\frac{2\mu + 4\lambda}{\sigma^2 + \lambda} - 1, \frac{\sigma^2 + \lambda}{2} \mid \frac{1}{w}\right), \quad (\text{eq.1})$$

where the expression $\text{GammaDist}(\cdot|\cdot)$ denotes the cumulative distribution function (CDF) of the Gamma distribution and the FOUR parameters $\{\mu, \sigma, \lambda, w\}$ capture the portfolio's expected return, investment volatility, (inverse of) life expectancy and initial nest egg, respectively. To be precise, in Microsoft Excel, the approximation can be implemented using the following syntax: $\text{GAMMADIST}(\text{spending rate as a fraction of initial wealth}, (2\mu + 4\lambda)/(\sigma^2 + \lambda) - 1, (\sigma^2 + \lambda)/2, \text{TRUE})$.

Exhibit #12 compares the results of this formula against more extensive simulations. For instance, if a 65 year-old withdraws \$6-per-\$100 adjusted for inflation for the rest of their life, the ERG formula states that there is a 26.96% chance that the money will run out before death. This result differs by roughly 5% when compared to extensive Monte Carlo Simulations under which the probability of ruin is 21.73% (using RP 2000 mortality tables). Exhibit #12 also summarizes simulation results under two other mortality assumptions, which are known in the actuarial literature as Gompertz and Exponential assumptions.

We believe that the ERG formula is simple and easy to use and can play a substantial role in demonstrating the link between spending rates, uncertain longevity and uncertain returns – all critical variables affecting retirement planning. A typical number for $\{\mu\}$ might be 7%, while $\{\sigma\}$ is 20%, and $\{\lambda\}$, which is a mortality rate, is 1/20 or 1/30, depending on age. Finally, $\{w\}$ is the ratio of the initial nest egg to desired spending. For example, $\{w = 100/5 = 20\}$.

For example, start with an investment (endowment, nest egg) fund containing \$20 that is invested in an equity fund that is expected to earn a continuously compounded return of $\{\mu = 7\%\}$ per annum, with a volatility or standard deviation of $\{\sigma = 20\%\}$ per annum. Assume that a 50-year-old early retiree with a median future lifespan of 27 years (according to actuarial mortality tables) intends on consuming \$1 after-inflation per annum for the rest of his or her life.

Note that if the median lifespan is assumed to be 27 years, then the probability of survival for 27 years is exactly 50%, which implies that the “mortality rate” parameter $\{\lambda = \ln[2]/27 = 0.0257\}$. According to (eq.1) (and after entering GAMMADIST(1/20, 2.695, 0.0328, TRUE) into Excel), the probability of retirement ruin is approximately 26%.

We believe that a simple formula like this can be of great pedagogical use in the field on retirement income planning. From a qualitative point of view, the bottom line is as follows: withdrawing more than (an inflation adjusted) 5% of initial capital annually, even under an aggressive asset allocation, appears excessively risky and could well prove to be the reason for a failed retirement plan.

The Critical Years

Clearly, earning a higher return across all retirement years, either because of good investment choices or plain old luck, results in a decreased probability of ruin. But which years are the most critical? The technique of Monte Carlo Simulations can again be of use. Exhibit #13 provides some insight into this issue. Recall from Exhibit #7 that if a 65 year-old male invests his entire nest egg in risky equities – which are expected to grow at 5% per annum -- and he

consumes an inflation-adjusted \$7 each year, the probability of retirement ruin is approximately 30%. However our simulations, as displayed in Exhibit #13, indicate that earning a 10% compound annual return each year during his first decade of retirement – i.e. between the ages of 65 and 75 - but allowing for random returns during his remaining lifetime, would reduce his probability of retirement ruin by roughly 23%, to 7%. Conversely, fixing the annual return at 0% for the first decade of retirement would increase the probability of ruin from 30% to 75%. This should make sense intuitively – better than expected returns lead to better odds and vice versa.

What is interesting, however, is that fixing the annual returns throughout the second and third decade of retirement – as opposed to the first – has a much smaller impact on the retirement ruin probability. Notice in Exhibit #13 that if the compound annual return during the second decade was "forced" to be 10% per annum, the retirement ruin probability is reduced to 15%, instead of 7%. And, if the compound annual return during the third decade was forced to be 10%, the probability drops to only 25%.

Clearly, it is much more favorable to earn the (abnormally) high return in the first decade of retirement, compared to the second or third decade of retirement. By symmetry, earning poor investment returns in the first decade, rather than second or third, exacerbates the retirement ruin probability. Whereas fixing a return of 1% for the first decade results in a ruin probability of nearly 70%, earning the same rate in the second decade results in lower ruin odds of about 50%, and fixing the rate in the third decade – about 35%.

In the technical language of probability theory, the conditional probability of retirement ruin is most sensitive to the returns over the first few years of retirement.

In sum, the main insight from this analysis is that the first decade of retirement is the most crucial one in determining whether your retirement plan will be successful. Thus attention must especially be focused on these years. This brings us to the role of product allocation for retirement risk management, which is quite different from the above-mentioned asset allocation.

The Role of Downside Protection

Since the first decade of retirement is the most crucial for achieving a sustainable financial plan, it makes sense to seek out financial products that provide downside protection during this particular period. In effect, this allows you or your client to increase his or her sustainable spending rate, but without taking on additional risk.

As Exhibits #11, #12 and #13 suggest, if a retiree decides to invest too heavily in risky equity, he or she runs a higher risk of retirement ruin if markets perform poorly during the first decade of retirement. Conversely, investing "too little" in the equity funds also heightens risk of ruin since the funds cannot grow sufficiently in order to support the chosen spending rate. It seems that you and your client are in a "damned if you do" and "damned if you don't" situation.

However, there is a third alternative, and that is to use derivatives – namely put and call options - to concentrate investment returns around a central value that, in most cases, will improve the sustainability of the portfolio. As a refresher of financial options, recall that these are derivative instruments whose value is derived from the value of some underlying investment such as a stock or bond. Whereas buying a call option grants an individual the right, but not the obligation to purchase an investment at a predetermined price, buying a put option guarantees the holder the right to sell the underlying investment at a predetermined price. A strategy we label a "retirement collar" involves selling a call with a strike price K_c and then using the proceeds to purchase a put with a

lower strike price K_p . Thus, if the asset's market price falls below K_p , you are guaranteed a minimum return since you have the right to sell it at a price of K_p . However, if the asset's value increases above a value of K_c , you will have to sell the asset to the holder of the call at a price of K_c , therefore, limiting the gains you could have earned on the portfolio.

Exhibit #14 provides an example of how this would work for a retirement portfolio. Imagine that at retirement you decide to allocate your \$100 nest egg (which can arbitrarily be scaled up or down) and consume \$4 per year from this nest egg. If all of the money is invested in equity-based products, the simulation results suggest that the probability of retirement ruin is 7.3% for a male (and 8.4% for a female, as per Exhibit #15). But, if you or your client purchase a 3-month put option that is 5% out-of-the-money, which means that the strike price is initially at \$95, and you fund this purchase by selling a call option that is 6.6% out of the money, the put/call combination will reduce the dispersion of your portfolio and will reduce the probability of ruin to 1.5% for a male and 2.4% for a female.

Essentially, we obtain this improvement in probability because we have removed the very large negative returns from all scenarios, thus increasing the chances that your initial nest egg is sufficient to maintain your desired standard of living. We emphasize, though, that this “collar strategy” is not a free lunch since large negative returns are reduced at the expense of reducing the upside potential of the portfolio. This is yet another manifestation of the universal trade-off between financial risk and return. Thus, although the portfolio's income will “last” longer by “delaying its date” with zero, the portfolio will not grow or increase in value as rapidly as the un-collared or unprotected portfolio. A sample path of the wealth in retirement with and without “collar protection” can be seen in Exhibit #16.

Exhibit#17 also illustrates a possible result of this strategy graphically. One line represents the expected value of wealth from time $t=0$ to $t=40$, assuming a 100% allocation to risky equity expected to earn 7% per annum with a standard deviation of 20%. The second line represents the expected value of wealth assuming the 100% equity allocation is protected with a collar, whose 3-month put option is 5% out of the money. Remember, this means that during any given quarter, the most a portfolio can lose is 5%. This put option is funded by selling a 3-month call option that is out of the money. Notice that, although both curves start-off at a value of 100, the expected level of wealth of the un-collared portfolio is uniformly higher throughout the 35 to 40 year horizon. Thus, the downside (variance or standard deviation) risk is reduced, but so is the upside potential.

Thus, although the portfolio protection comes at a price, it has the potential to play a critical role in mitigating the risk of retirement ruin, especially if used in the early years of retirement. The underlying logic of a “trading upside participation for downside protection” strategy is at the heart of the innovative riders and guarantees that have recently been attached to variable annuity (VA) policies. Indeed, guaranteed minimum withdrawal benefits (a.k.a. GMWBs) extend the natural lifespan of a portfolio by implementing a similar tradeoff.

Why Longevity Insurance?

As we have discussed earlier, maintaining a balanced asset allocation, purchasing downside protection in the form of option guarantees and selecting a sensible consumption rate helps to improve the odds that a given retirement plan will be financially sustainable. However, the probabilities of retirement ruin under many scenarios cannot be completely eliminated, and this risk should be managed using longevity insurance – namely, payout annuities.

It is a fact that most retirees are hesitant to annuitize their variable annuity (VA) contracts – or for that matter to voluntarily purchase immediate annuities with their IRA, 401(k) and other liquid wealth – because they fear losing control and/or believe they can “do better” with other investment alternatives. Oddly enough, when people within a traditional Defined Benefit (DB) pension plan are coaxed to “switch into” a money-purchase Defined Contribution (DC) pension plan and give-up the implicit life annuity, most turn the offer down, while others react violently and litigiously. It seems there is a fog of confusion surrounding the financial benefits from annuitization. So, motivated by the need to *keep it simple*, in this section we describe a simple story that can illustrate the pedagogical benefits of annuitization and longevity insurance, while positioning the product firmly within the realm of investment risk and return.

Imagine a 95 year-old retiree who loves playing bridge with her four best friends on Sunday every few months. Coincidentally, all five of them are exactly 95 years old, quite healthy, and have actually been retired – and playing bridge -- for 30 years. Recently this game has become, somewhat tiresome, and one of them has decided to juice-up their activities. Last time they met, she proposed that they each take \$100 out of their wallets and place the money on the kitchen table. “Whoever survives to the end of the year gets to split the \$500...” she said. “And, if you don’t make it, you forfeit the money.” Yes, this is an odd gamble, but you will see our point in a moment.

Imagine that all five women thought it was an interesting idea and agreed, but felt it was risky to keep \$500 on the kitchen table for a whole year. So, they decided to put the money in a local bank’s one-year certificate of deposit, paying 5% interest for the year.

So what will happen next year? According to statistics compiled by actuaries at the U.S. Social Security administration, there is a 20% chance that any given member of the bridge club will pass-on to the next world during the

next year. This, in turn, implies an 80% chance of survival. And, while virtually anything can happen during the next 12 months of waiting – actually, there are 120 combinations, believe it or not -- the odds imply that *an average* four 96-year olds will survive to split the \$525 pot at year-end.

Note that each survivor will get \$131.25 as their total return on the original investment of \$100. The 31.25% investment return contains 5% of the bank's money and a healthy 26.25% of "mortality credits". These credits represent the capital and interest "lost" by the deceased and "gained" by the survivors.

The catch, of course, is that the average non-survivor forfeited their claim to the funds. And, while the beneficiaries of the non-survivor might be frustrated with the outcome, the survivors get a superior investment return. More importantly, they ALL get to manage their *lifetime income risk* in advance, without having to worry about what the future will bring.

We think this story does a nice job of translating the benefits of longevity insurance into investment rates of return. Personally, I find no other financial product that guarantees such high rates of return, conditional on survival.

In fact, this story can be taken one step further. What if the club decided to invest the \$500 in the stock market, or some risky NASDAQ high-tech fund, for the next year? Moreover, what happens if this fund or sub-account collapses in value during the next year and falls 20% in value? How much will the surviving bridge players lose? Well, if you are thinking "nothing", that is absolutely the correct answer. They divide the \$400 amongst the surviving four and get their original \$100 money back.

Such is the power of mortality credits. They subsidize losses on the downside and enhance gains on the up-side. In fact, we would go so far as to say that once you wrap true longevity insurance around a diversified portfolio, the annuitant can actually afford and tolerate more financial risk.

Of course, real live annuity contracts do not work in the way described above. The “tontine” contract is renewable each year and the surviving 96-year olds have the option to take their mortality credits and go home. In practice, annuity contracts are for life, and these credits are spread and amortized over many years of retirement. But the basic *insurance economics* underlying the contract are exactly as described above.

A natural question to ponder is whether this life-roulette game would yield such high returns at younger ages, and the answer is no. Exhibit #18 provides a rough estimate of the relative magnitude from annuitizing – with true life annuities -- at different ages. At age 55 the mortality credits are less than one percent, or a mere 55 basis points. At age 65 the number increases to 83 basis points, which is still nothing to get excited about. To put these numbers in context, if a recent retiree or their advisor thinks they can earn 83 basis points more than the pricing rate used by the annuity vendor, they are better-off not annuitizing today and managing the money with a systematic withdrawal plan themselves. This “benchmark” or “hurdle” rate of return can be used to assess the relative benefits from annuitization at different ages. Note that by the mid 80s it becomes virtually impossible to beat – what we like to call -- the Implied Longevity Yield¹. To put it crudely, too many are people are dying. In fact, we find that the best deal can be obtained by acquiring longevity insurance at a relatively young age – mid 40s for example – but to have it start paying income in the late 70s. For more information on the design of these contracts see Milevsky (2005a).

In sum, pension/life annuities provide a very unique and peculiar kind of insurance. It is virtually the only insurance policy that people acquire during the course of their life, but actually hope to use! While we are all willing to pay for home insurance, disability insurance or car insurance, we never actually want to

¹ The "Implied Longevity Yield" and its acronym - "ILY" are registered trade-marks and are the property of CANNEX Financial Exchanges

exercise or use the policy. After all, who wants their house to burn down, leg to break or car to crash, G-D forbid. Yet, the “insurable event” underlying pension annuities is living a long and prosperous life. Perhaps this is why the industry has yet to achieve the level of success in marketing and selling these products – they are still accustomed to scaring us. Hopefully, simple tales like the above can help retirees and their financial advisors understand the benefits, risks and returns from buying longevity insurance.

Conclusion and Main Points:

This chapter – which was partially motivated by the tragicomic story of Jorge Guinle -- has made a number of arguments that can all be tied together under the label of financial risk management during retirement:

1. A 25 year retirement is quite feasible and highly probable for today's generation of baby boomers. Indeed, at least one member of a couple aged 65 will reach age 90. At the same time, there is also chance that the couple will live well-into their 90s, and a chance they will not even reach age 80. Either way, this longevity risk – and it is a financial risk – can not be ignored. Like life insurance, car insurance and home insurance, this pure risk should be hedged and insured like all other risks we face during our human lifecycle.
2. Retirees face a unique type of inflation risk which is confirmed by the design of the Bureau of Labor Statistics' CPI-Elderly index. Furthermore, we have argued that, after the above-mentioned 25 years of retirement, a steady source of (nominal) income will likely be worth a third to a half less than it was at the time of retirement.
3. We confirm the results of many similar studies: from a statistical perspective there is very little justification for spending more than 5% of initial capital, adjusted for inflation, during retirement. Whether relying on the results of Monte Carlo Simulations (MCS) under varying mortality and

asset-dynamic assumptions, or on the Exponential Reciprocal Gamma (ERG) approximation which was developed in Milevsky and Robinson (2005), the probability of ruin during retirement will exceed 15% if a higher than 5% consumption rate is planned on.

4. The first seven to ten years of retirement is the most critical investment period in one's life, with returns that are highly correlated with the probability of a sustainable retirement. For example, if a retiree consuming an inflation-adjusted 5% earns a compound annual return of only 1% in the first ten years, as opposed an assumed growth rate of 5% for the portfolio, the probability of retirement ruin would jump from about 30% to nearly 70%.
5. Our statistical results lead to a very strong case for using non-linear instruments, such as put options and other downside protection, to limit the impact of a catastrophic first decade of retirement. Insurance products that provide downside protection – even at the expense of upside participation – will reduce the probability of retirement ruin and hence increase the sustainability of the portfolio.
6. Finally, we lament the lack of popularity or voluntary purchases of individual pension annuities which are the foundation of Defined Benefit (DB) pensions. We have shown that the implicit rate of return on longevity insurance – and by extension, pension annuities – can reach 20% and even higher, as people age. Thus, there are substantial benefits to annuitization and this should be encouraged as a risk management strategy.

References

Ameriks, J., M. Veres and M. Warshawsky (2003), Retirement income that lasts a lifetime, *Journal of Financial Planning*, pg 20-30.

Blake, D., Cairns, A.J., and Dowd, K. (2000). PensionMetrics: Stochastic pension plan design during the distribution phase, Pensions Institute Working Paper.

Chen, P. and Milevsky, M.A. (2003), Merging asset allocation and longevity Insurance: an optimal perspective on payout annuities, *Journal of Financial Planning*, Vol. 16(6): 52-62.

Ho, K., M. Milevsky and C. Robinson (1994), How to avoid outliving your money, *Canadian Investment Review*, Vol. 7(3), pg. 35-38.

Ibbotson, Chen and Henkel (2005), Longevity risk, chapter in this book.

Ibbotson Associates. Stocks, Bonds, Bills and Inflation 2004 Yearbook. Chicago: *Ibbotson Associates*, 2004.

Milevsky, M.A. (2005a), Real Longevity Insurance with a Deductible: Introduction to Advanced Life Delayed Annuities (ALDA), *North American Actuarial Journal*, forthcoming.

Milevsky, M.A. (2005b) *The Calculus of Retirement Income: Financial Models for Pensions and Insurance*, Cambridge University Press, forthcoming.

Milevsky, M.A. and C. Robinson (2005), What is a sustainable spending rate? A simple answer (that doesn't require simulation), *Financial Analysts Journal*, forthcoming.

Pye, G. (2000), Sustainable investment withdrawals, *Journal of Portfolio Management*, Vol. 26(3), pg. 13-27.

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Exhibit #1

Conditional Probability of Survival at Age 65

| To Age: | Female | Male |
|----------------|---------------|-------------|
| 70 | 93.9% | 92.2% |
| 75 | 85.0% | 81.3% |
| 80 | 72.3% | 65.9% |
| 85 | 55.8% | 45.5% |
| 90 | 34.8% | 23.7% |
| 95 | 15.6% | 7.7% |
| 100 | 5.0% | 1.4% |

Source: Society of Actuaries RP-2000 Table with full projection.

Exhibit #2

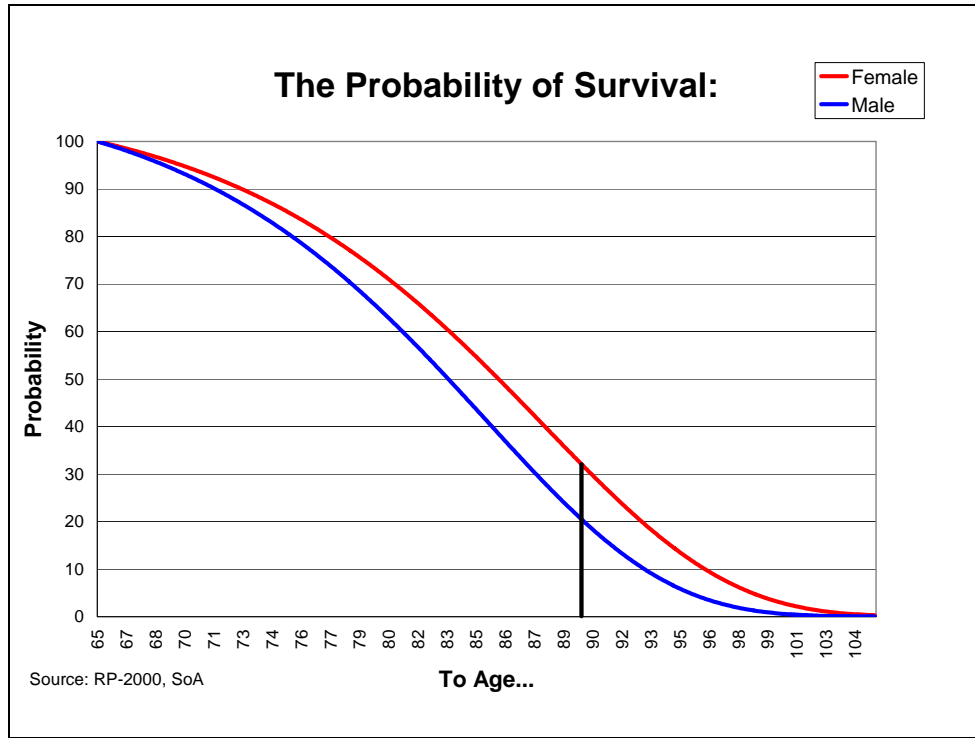


Exhibit #3

A Couple Aged 65....Survival:

| To Age: | At Least One... |
|---------|-----------------|
| 70 | 99.5% |
| 75 | 97.2% |
| 80 | 90.6% |
| 85 | 75.9% |
| 90 | 50.3% |
| 95 | 22.1% |

Source: Society of Actuaries RP-2000 Table (with full projection)

Exhibit #4

Inflation: What Does a \$1,000 Payment, Really Buy You?

| Years in Future | Realized inflation rate during each year.... | | | | | | |
|--------------------|--|-------|-------|-------|-------|-------|-------|
| | 0% | 1% | 2% | 4% | 6% | 8% | 10% |
| 1 | \$1,000 | \$990 | \$980 | \$962 | \$943 | \$926 | \$909 |
| 5 | \$1,000 | \$952 | \$906 | \$822 | \$747 | \$681 | \$621 |
| 10 | \$1,000 | \$905 | \$820 | \$676 | \$558 | \$463 | \$386 |
| 15 | \$1,000 | \$961 | \$743 | \$555 | \$417 | \$315 | \$239 |
| 20 | \$1,000 | \$820 | \$673 | \$456 | \$312 | \$215 | \$149 |
| 25 | \$1,000 | \$780 | \$610 | \$375 | \$233 | \$146 | \$92 |
| 30 | \$1,000 | \$742 | \$552 | \$308 | \$174 | \$99 | \$57 |
| 35 | \$1,000 | \$706 | \$500 | \$253 | \$130 | \$68 | \$36 |

Exhibit #5

Inflation: Relative Weights

| Category | CPI-W | CPI-E |
|-----------------|--------------|--------------|
| Food: | 17.9% | 14.3% |
| Housing: | 37.6% | 45.9% |
| Apparel: | 3.9% | 2.8% |
| Transport: | 18.8% | 13.8% |
| Medical Care: | 5.1% | 10.2% |
| Recreation | 5.3% | 4.4% |
| Education | 5.1% | 2.9% |
| Other | 6.1% | 5.6% |

Source: U.S. Department of Labor, Bureau of Labor Statistics

Exhibit #6

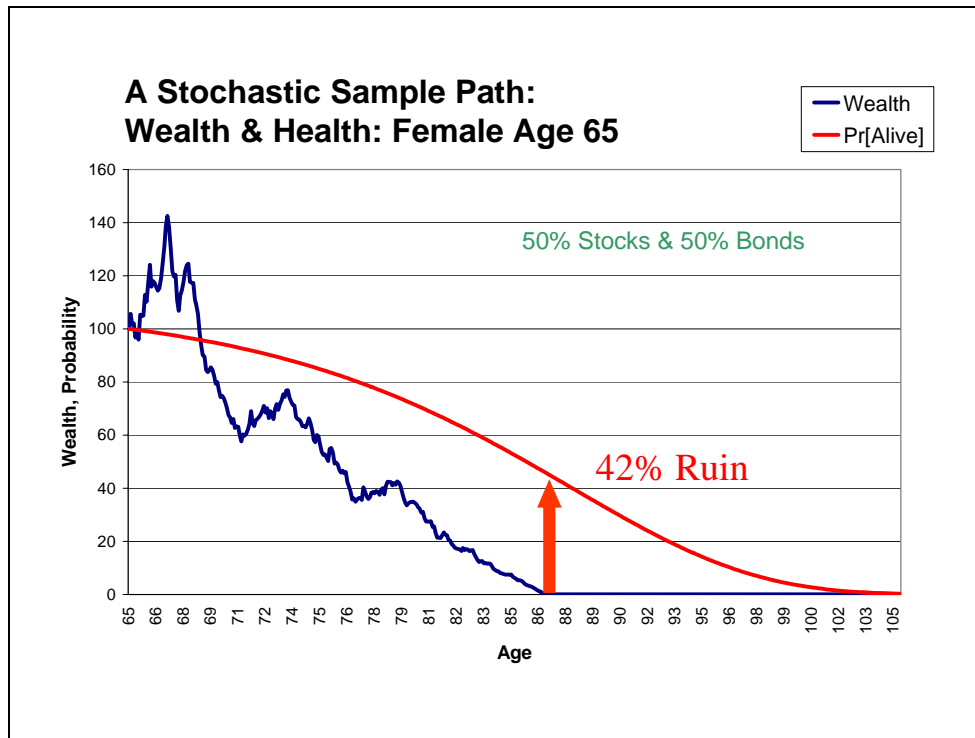


Exhibit #7

Probability of Retirement Ruin:
Assuming **Fixed Real** Consumption

| Male 65 | \$4 c. | \$5 c. | \$6 c. | \$7 c. | \$8 c. |
|----------------|---------------|---------------|---------------|---------------|---------------|
| 0% Eq. | 8.3% | 19.9% | 33.3% | 45.8% | 56.0% |
| 25% Eq. | 4.2% | 12.8% | 24.8% | 37.8% | 49.3% |
| 50% Eq. | 3.8% | 10.6% | 20.4% | 31.8% | 42.9% |
| 75% Eq. | 5.0% | 11.4% | 19.8% | 29.4% | 39.0% |
| 100% Eq. | 7.3% | 13.7% | 21.3% | 29.7% | 37.9% |

Exhibit #8

Probability of Retirement Ruin:
Assuming **Fixed Real** Consumption

| Female 65 | \$4 c. | \$5 c. | \$6 c. | \$7 c. | \$8 c. |
|------------------|---------------|---------------|---------------|---------------|---------------|
| 0% Eq. | 9.9% | 23.1% | 37.4% | 50.3% | 63.3% |
| 25% Eq. | 5.1% | 15.0% | 28.2% | 41.9% | 53.6% |
| 50% Eq. | 4.5% | 12.3% | 23.1% | 35.3% | 46.7% |
| 75% Eq. | 5.9% | 13.0% | 22.2% | 32.4% | 42.4% |
| 100% Eq. | 8.4% | 15.4% | 23.5% | 32.3% | 40.9% |

Exhibit #9

Probability of Retirement Ruin:
Assuming **Fixed Real** Consumption

| Male 75 | \$4 c. | \$5 c. | \$6 c. | \$7 c. | \$8 c. |
|----------------|---------------|---------------|---------------|---------------|---------------|
| 0% Eq. | 1.3% | 4.5% | 9.9% | 17.0% | 24.6% |
| 25% Eq. | 0.5% | 2.6% | 6.7% | 12.8% | 20.0% |
| 50% Eq. | 0.6% | 2.4% | 5.8% | 11.1% | 16.7% |
| 75% Eq. | 1.1% | 3.1% | 6.5% | 11.2% | 16.7% |
| 100% Eq. | 2.0% | 4.5% | 8.0% | 12.5% | 17.6% |

Exhibit #10

**Probability of Retirement Ruin:
Assuming **Fixed Real** Consumption**

| Female 75 | \$4 c. | \$5 c. | \$6 c. | \$7 c. | \$8 c. |
|------------------|---------------|---------------|---------------|---------------|---------------|
| 0% Eq. | 2.9% | 8.5% | 16.6% | 25.7% | 34.6% |
| 25% Eq. | 1.4% | 5.2% | 11.6% | 20.1% | 29.1% |
| 50% Eq. | 1.4% | 4.5% | 9.8% | 17.1% | 25.2% |
| 75% Eq. | 2.1% | 5.4% | 10.3% | 16.6% | 23.7% |
| 100% Eq. | 3.5% | 7.1% | 12.0% | 17.8% | 24.0% |

Exhibit #11

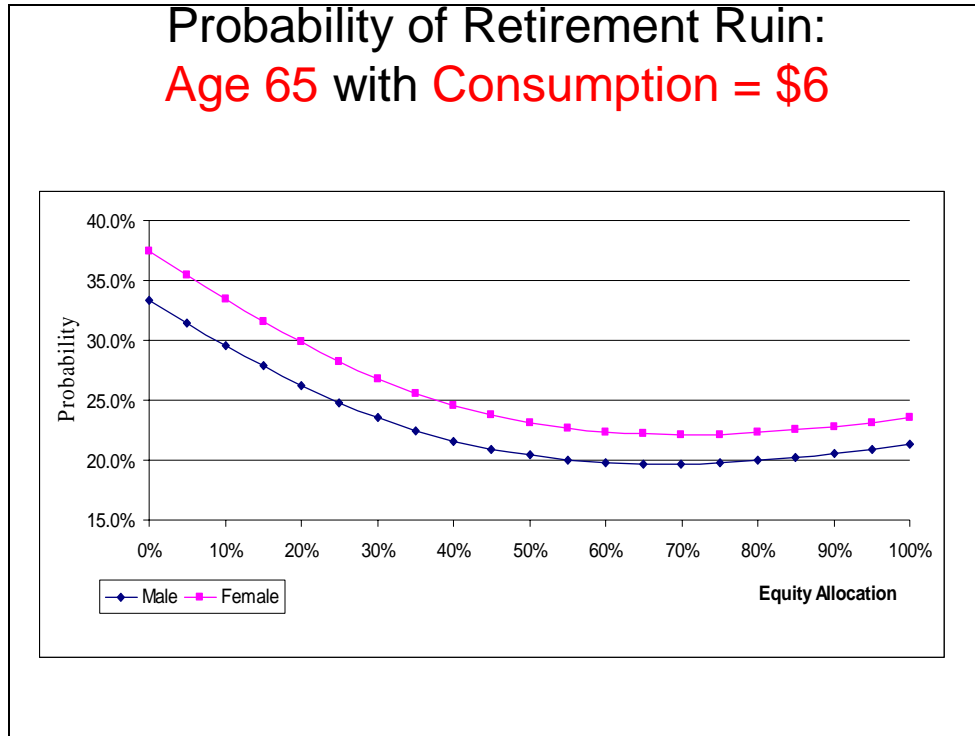


Exhibit # 12

How good is the simple (ERG) formula for predicting retirement ruin?

| Withdrawal (per initial \$100) | Probability Your Retirement Nest Egg is Not Enough to Sustain Your Retirement | | | |
|--------------------------------------|--|----------|-------------|---------------|
| | RP 2000 | Gompertz | Exponential | ERG Approx |
| \$2 | 0.32% | 0.30% | 1.05% | 2.76% |
| \$3 | 1.93% | 2.22% | 4.44% | 6.94% |
| \$4 | 5.94% | 6.16% | 10.60% | 12.69% |
| \$5 | 12.51% | 12.70% | 17.70% | 19.53% |
| \$6 | 21.73% | 22.27% | 28.44% | 26.96% |
| \$7 | 32.00% | 31.98% | 37.42% | 34.59% |
| \$8 | 42.935% | 42.92% | 48.18% | 42.10% |

Note: Portfolio is assumed to earn 7% (inflation adjusted) in any given year, with a volatility (uncertainty, risk) of 20%. This leads to a (geometric mean) growth rate of 5%

Exhibit #13

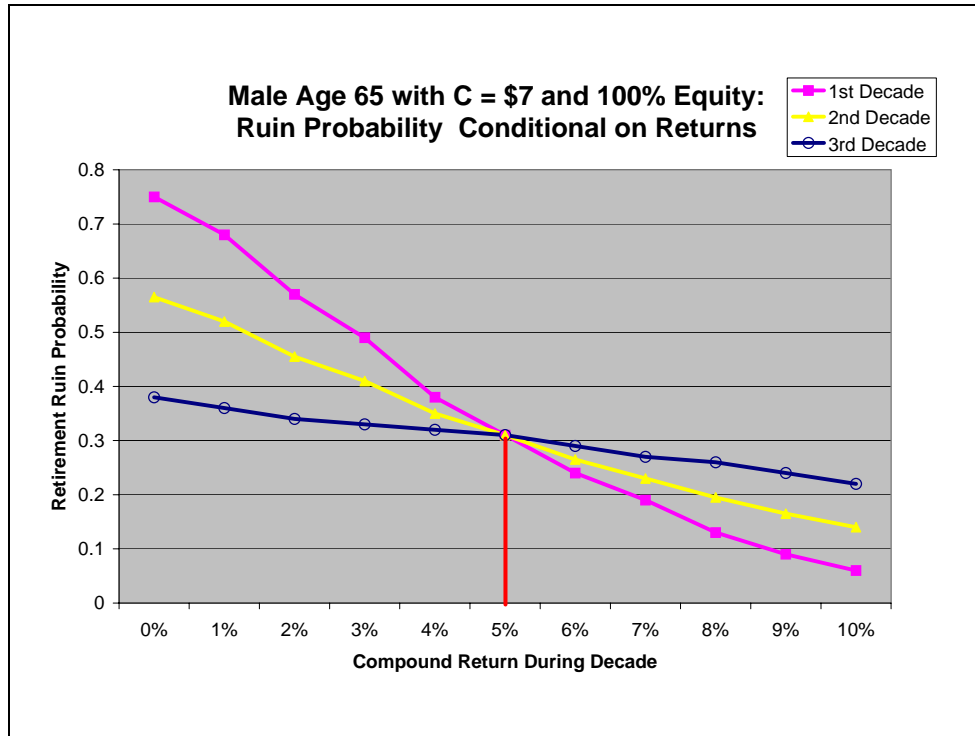


Exhibit #14

Probability of Retirement Ruin: Under a fixed spending rate...

| MALE 65 | 4 Dollars | 5 Dollars | 6 Dollars |
|--------------------------|-------------|--------------|--------------|
| No Downside Protection | 7.3% | 13.7% | 21.3% |
| - 5% against + 6.6% | 1.5% | 6.0% | 16.8% |
| - 10% against + 12.8% | 4.1% | 9.7% | 18.3% |

Exhibit #15

Probability of Retirement Ruin:
Under a fixed spending rate...

| FEMALE 65 | 4 Dollars | 5 Dollars | 6 Dollars |
|--------------------------|-------------|--------------|--------------|
| No Downside Protection | 8.4% | 15.4% | 23.5% |
| - 5% against + 6.6% | 2.4% | 10.5% | 24.0% |
| - 10% against + 12.8% | 5.9% | 14.1% | 25.1% |

Exhibit #16

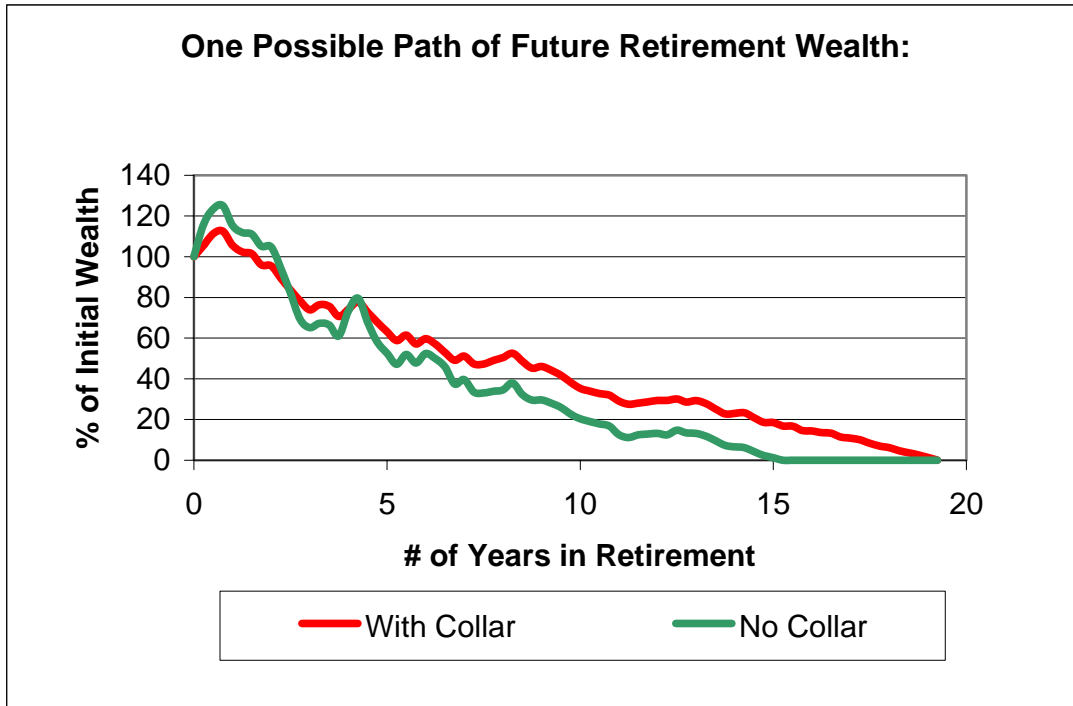


Exhibit #17

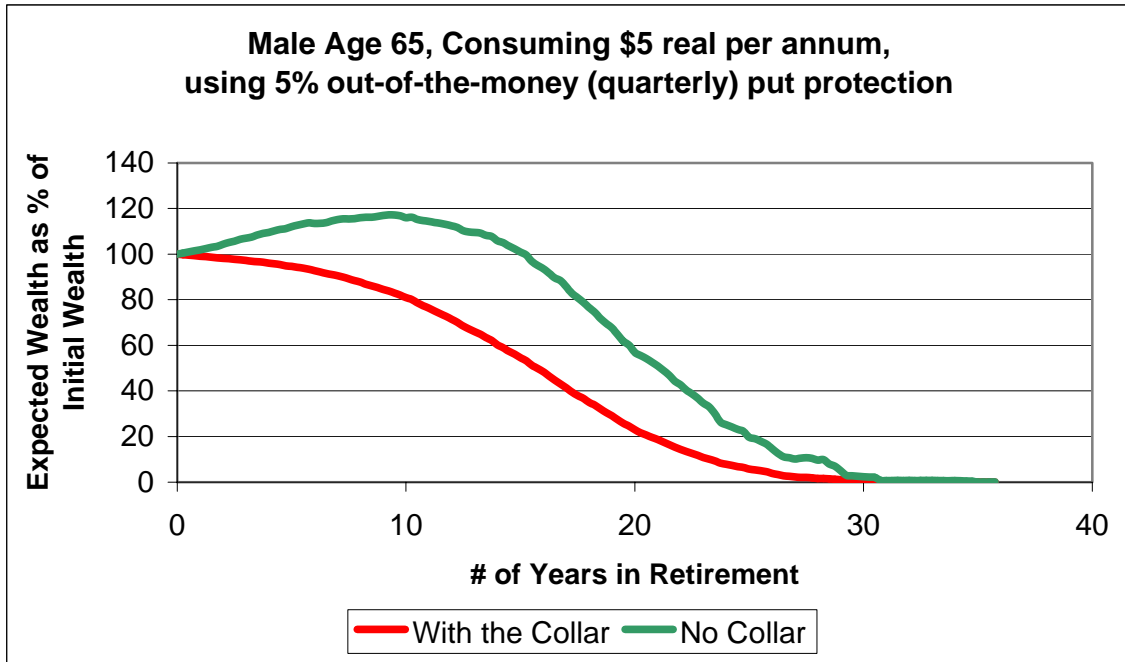


Exhibit #18

Value of Unisex Mortality Credits:
 What must you earn -- above the pricing
 rate -- to justify **NOT** annuitizing?

| Age of Annuitant | Spread Above Pricing Interest Rate <small>(in Basis Points = 1/100 %)</small> |
|------------------|--|
| 55 | 35 |
| 60 | 52 |
| 65 | 83 |
| 70 | 138 |
| 75 | 237 |

| Age of Annuitant | Spread Above Pricing Interest Rate <small>(in Basis Points = 1/100 %)</small> |
|------------------|--|
| 80 | 414 |
| 85 | 725 |
| 90 | 1256 |
| 95 | 2004 |
| 100 | 2978 |

Source: The IFID Centre calculations
 Assuming 40m/60f (static) Annuity 2002 Table at 6% net interest.